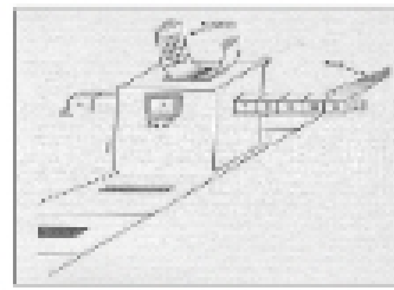
CS 302 Lecture 15
Turing Machine Robustness; Nondeterministic Turing Machines; Unrestricted Grammars

Qi Mi
Computer Science Department
University of Virginia

Recap the Turing machines

- 7-tuple: $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$



- Possible modifications to Turing machines?

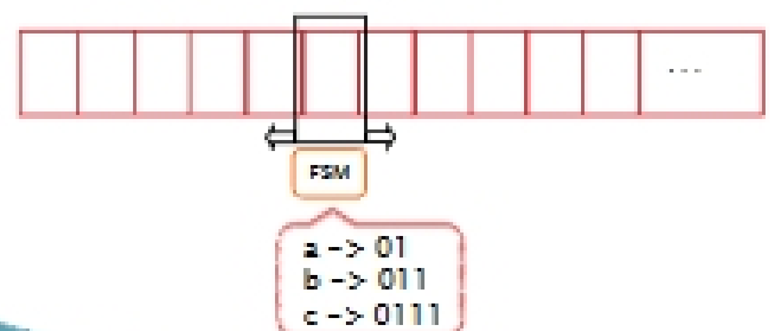
TM with a different alphabet size

- Consider a Turing machine with an input alphabet of {a, b, c} and another with an input alphabet of {0, 1}. Which is more powerful?

$|\Sigma|$

TM with a different alphabet size

- Idea: Use FSM to translate the encoding between different alphabets.



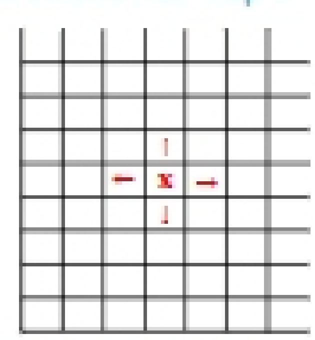
TM with a different alphabet size

- Encoding
  - A process of transforming information from one format into another without loss of information.
- Example:
  - Binary representation of numbers
- Application:
  - Adding marker symbols that are not in the original alphabet when you design a TM will not change the power of TM.

TM with a multidimensional tape

- A 2-dimensional tape

  {L, R, U, D}

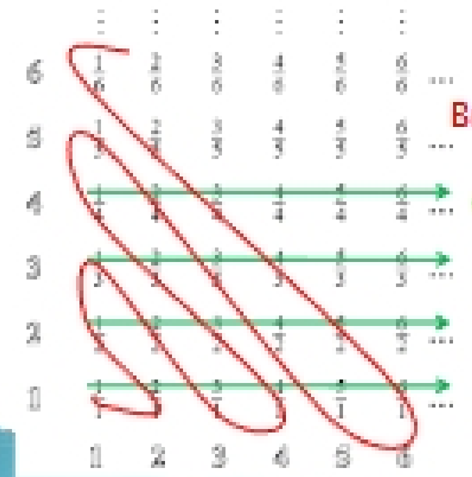


- Question: Is a TM with a 2-dimensional tape equivalent to one with an ordinary 1-dimensional tape?

TM with a multidimensional tape

- *The set of rational numbers:*
  Q = { p/q | p and q are natural numbers and co-prime}
- *The set of natural numbers:*
  N = {1, 2, 3, 4, 5, …}
- True or False: |Q| > |N|?

TM with a multidimensional tape



|Q| = |N|

Breadth-first search, instead of depth-first search

Dovetailing

TM with a multidimensional tape

- Question: Recall that adjacent cells may become non-adjacent when we map a 2-dimensional tape to a 1-dimensional tape. How do we solve the issue of mapping the head movement between adjacent cells on a 2-dimensional tape to that on a 1-dimensional tape?

TM with a multidimensional tape

- Map a 2-dimensional tape to an ordinary 1-dimensional tape.
- Map a k-dimensional tape to an ordinary 1-dimensional tape.
- Summary:
  - Dovetailing (interleaving)
  - Mapping (1-to-1 correspondence)

Turing machine modifications

- A different alphabet size
- Multidimensional tape
- Doubly-infinite tape
- Multiple tapes
- Etc

Theorem: All these modifications do NOT increase the power of TM's. -- TM robustness

Question: What if a combination of the above?

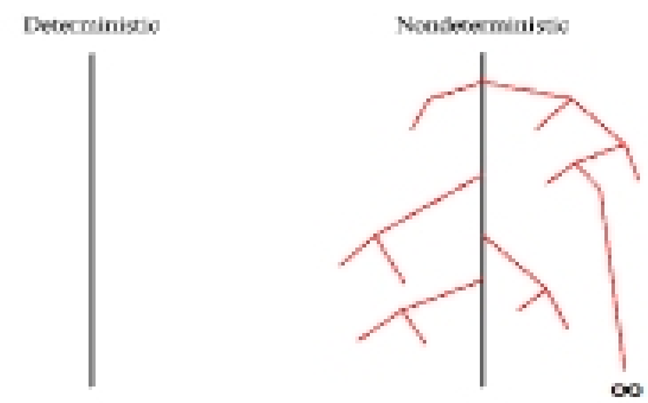
Designing Turing machines

- Task: Design a Turing machine that can recognize $\{ww | w \in \Sigma^*\}$?

Non-deterministic Turing machines

- A Turing machine is deterministic if:
  $\forall\ q\in Q,\ a\in\Gamma\quad |\delta(q,a)| \le 1$
  i.e., no multiple choices allowed
- Otherwise, it is non-deterministic.
- A non-deterministic TM (NDTM) can have several choices of which state to proceed next in a computation.
- Many "next-moves":
  $\delta: Q\times\Gamma \rightarrow 2^{Q\times\Gamma\times\{L,R\}}$

Non-deterministic Turing machines



Designing Turing machines

Designing Turing machines

Non-deterministic Turing machines

- Question: Is the set of languages that can be decided by NDTM's larger than that by DTM's?

Non-deterministic Turing machines

- Simulate any non-deterministic TM N with a deterministic TM D.
- Three tapes: input tape, simulation tape, and address tape
- Have D try all possible branches of N using breadth-first search. (can't use depth-first search here)
- Conclusion: NDTMs and DTMs are equivalent in power.