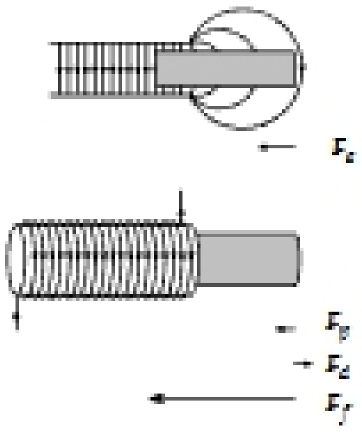
Today in Physics 217: magnetism in matter

- ❑ Magnetism
- ❑ Magnetization and bound currents
- ❑ Ampère's Law and magnets
- ❑ Linear magnetic media
- ❑ Cautions about H
- ❑ Example: H and Ampère's Law



Magnetism

There are three forms of magnetism in matter, all having to do with **spins of atomic electrons**.

- ❑ Electrons have spin, and spinning charges have magnetic dipole moments that can be aligned by external magnetic fields.
- ❑ However, the spin of electrons, and the pairings of electrons within atoms, are manifestations of **quantum-mechanical** effects, so magnetism is not really understandable simply from our present **classical** viewpoint.

Anyway, here are the three kinds of magnetism:

Magnetism (continued)

- ❑ **Ferromagnetism**
  The layman's magnetism, this is the only *strong* form of magnetism.
  - It comes from aligned spins of unpaired d electrons in transition metals, especially nickel, iron and cobalt (hence the name), and rare earths like samarium.
- ❑ **Paramagnetism**
  Analogous to electric polarization, and much weaker than ferromagnetism; can almost understand classically.
  - Produced by interaction of B with dipole moments of unpaired s or p electrons. Aluminum, for example, has one p electron in its valence shell, and is paramagnetic. So is molecular oxygen, which has two unpaired spins among its bonding electrons.

Magnetism (continued)

- **Diamagnetism**
  Comes from the interaction of B with induced magnetic dipoles in atoms; also much weaker than ferromagnetism.
  - Contrary to the way electric polarization works, diamagnetism is characterized by magnetization in the direction *opposite* that of the applied field.
  - Seen best in materials in which all the electrons are paired off (Ne, N_2, ...).

Whatever the type, we can define a **magnetization**, in analogy with the electric polarization P:

$$M = \frac{\text{magnetic dipole moment}}{\text{unit volume}}$$

Magnetization and bound currents

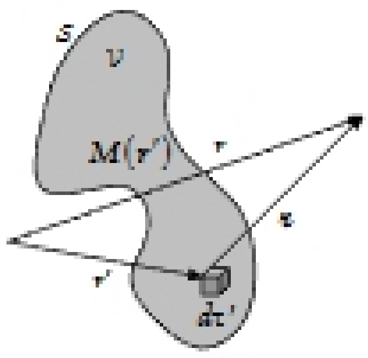
Like P, which we interpret in terms of bound charge, M can be interpreted in terms of bound currents, which we can characterize by consideration of the magnetic potential for a dipole.

$$A = \frac{m \times \hat{\mathbf{r}}}{\mathbf{r}^2} = \int_V \frac{M \times \hat{\mathbf{r}}}{\mathbf{r}^2} d\tau'$$

$$= \int_V M \times \nabla'\left(\frac{1}{\mathbf{r}}\right) d\tau'$$

Use Product Rule #7:

$$= \int_V \left[\frac{1}{\mathbf{r}} \nabla' \times M + \nabla' \times \left(\frac{M}{\mathbf{r}}\right)\right] d\tau'$$



Magnetization and bound currents (continued)

Now, for any vector function $v = v(r)$ and a constant vector C, we can write the divergence theorem as

$$\int \nabla \cdot (v \times C) d\tau = \oint (v \times C) \cdot da$$

$$\int \left[C \cdot (\nabla \times v) + v \cdot \overbrace{\cancel{(\nabla \times C)}}^{0}\right] d\tau = \oint (v \times C) \cdot da \qquad \text{Product Rule \#6}$$

$$C \cdot \int \nabla \times v d\tau = \oint (v \times C) \cdot da \qquad C \text{ is constant}$$

$$= C \cdot \oint da \times v \qquad \text{triple product rule \#1}$$

$$\int \nabla \times v d\tau = \oint da \times v \quad .$$

$$\text{Thus} \quad A = \int_V \frac{1}{\mathbf{r}} \nabla' \times M d\tau' + \oint_S \frac{M \times da'}{\mathbf{r}} \quad .$$

Magnetization and bound currents (continued)

Thus
$$A = \int_V \frac{1}{r}\nabla' \times M d\tau' + \oint_S \frac{M \times da'}{r}$$
$$= \frac{1}{c}\int_V \frac{1}{r} J_b d\tau' + \frac{1}{c}\oint_S \frac{1}{r} K_b da' \quad ,$$

where we have defined the bound current densities:
$$J_b(r') = c\nabla' \times M(r') \quad , \qquad (\text{cf. } \rho_b = -\nabla \cdot P,$$
$$K_b(r') = cM \times \hat{n}\big|_S \quad . \qquad \sigma_b = P \cdot \hat{n}\big|_S)$$

[In MKS, $J_b(r') = \nabla' \times M(r') \quad , \quad K_b(r') = M \times \hat{n}\big|_S \quad ,$
$$A = \frac{\mu_0}{4\pi}\int_V \frac{1}{r} J_b d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{1}{r} K_b da' \quad .]$$

Ampère's Law revisited

With bound currents we can do to Ampère's Law about what we did to Gauss's law before:

$$\nabla \times B = \frac{4\pi}{c} J = \frac{4\pi}{c}\left(J_{free} + J_b\right) = \frac{4\pi}{c}\left(J_f + c\nabla \times M\right)$$
$$\nabla \times \left(B - 4\pi M\right) = \frac{4\pi}{c} J_f$$
$$\Rightarrow \quad \nabla \times H = \frac{4\pi}{c} J_f \quad , \quad H \equiv B - 4\pi M \quad (\text{cf. } D = E + 4\pi P)$$

Or, in MKS, $\nabla \times H = J_f \quad , \quad H \equiv \frac{1}{\mu_0} B - M \quad .$

Linear magnetic media

As before, when the definition of D led to that of the electric susceptibility χ_e and the dielectric constant ε, we can define

$\cancel{M = \chi_m B \quad (\text{cf. } P = \chi_e E)}$ That would be too sensible.

$$M = \chi_m H$$

In these terms,
$$B = H + 4\pi M = H\left(1 + 4\pi\chi_m\right) \equiv \mu H \qquad (\text{cf. } D = \varepsilon E).$$

or, in MKS,
$$B = \mu_0\left(H + M\right) = \mu_0\left(1 + \chi_m\right) H \equiv \mu H \quad .$$

κ is called the **relative permeability**. Contrary to ε, κ is the same in cgs and MKS, though χ_m, equally dimensionless, is not.