Chapter 5. Magnetostatics

5.1. The Magnetic Field

Consider two parallel straight wires in which current is flowing. The wires are neutral and therefore there is no net electric force between the wires. Nevertheless, if the current in both wires is flowing in the same direction, the wires are found to attract each other. If the current in one of the wires is reversed, the wires are found to repel each other. The force responsible for the attraction and repulsion is called the **magnetic force**. The magnetic force acting on a moving charge q is defined in terms of the **magnetic field**:

$$\overline{F}_{magnetic} = q\left(\overline{v} \times \overline{B}\right)$$

The vector product is required since observations show that the force acting on a moving charge is perpendicular to the direction of the moving charge. In a region where there is an electric field and a magnetic field the total force on the moving force is equal to

$$\overline{F}_{total} = \overline{F}_{electric} + \overline{F}_{magnetic} = q\overline{E} + q\left(\overline{v} \times \overline{B}\right)$$

This equation is called the **Lorentz force law** and provides us with the total electromagnetic force acting on q. An important difference between the electric field and the magnetic field is that the electric field does work on a charged particle (it produces acceleration or deceleration) while the magnetic field does not do any work on the moving charge. This is a direct consequence of the Lorentz force law:

$$dW_{magnetic} = \overline{F}_{magnetic} \bullet d\overline{l} = q\left[\left(\overline{v} \times \overline{B}\right) \bullet \overline{v}\right] dt = 0$$

We conclude that the magnetic force can alter the direction in which a particle moves, but can not change its velocity.

Example: Problem 5.1

A particle of charge q enters the region of uniform magnetic field $\overline{B}$ (pointing into the page). The field deflects the particle a distance d above the original line of flight, as shown in Figure 5.1. Is the charge positive or negative? In terms of a, d, B, and q, find the momentum of the particle.

In order to produce the observed deflection, the force on q at the entrance of the field region must be directed upwards (see Figure 5.1). Since direction of motion of the particle and the direction of the magnetic field are known, the Lorentz force law can be used to determine the direction of the magnetic force acting on a positive charge and on a negative charge. The vector product between $\overline{v}$ and $\overline{B}$ points upwards in Figure 5.1 (use the right-hand rule). This shows that the charge of the particle is positive.

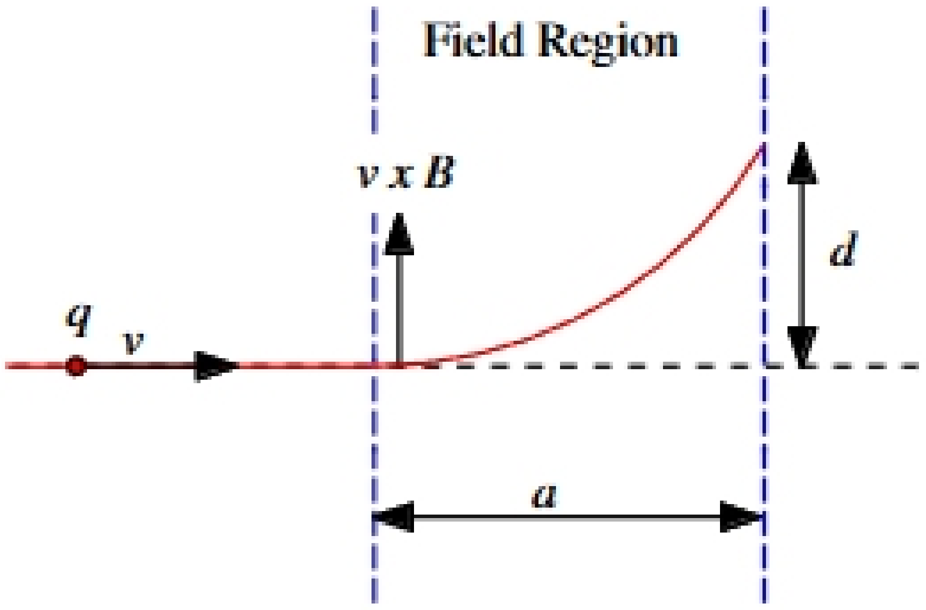

Figure 1. Problem 5.1.

The magnitude of the force acting on the moving charge is equal to

$$F_{magnetic} = qvB$$

As a result of the magnetic force, the charged particle will follow a spherical trajectory. The radius of the trajectory is determined by the requirement that the magnetic force provides the centripetal force:

$$F_{cent} = \frac{mv^2}{r} = F_{magnetic} = qvB$$

In this equation r is the radius of the circle that describes the circular part of the trajectory of charge q. The equation can be used to calculate r:

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

where p is the momentum of the particle. Figure 5.2 shows the following relation between r, d and a:

$$(r-d)^2 + a^2 = r^2$$

This equation can be used to express r in terms of d and a:

$$r = \frac{d^2 + a^2}{2d}$$

The momentum of the charge q is therefore equal to

$$p = qBr = \frac{d^2 + a^2}{2d} qB$$

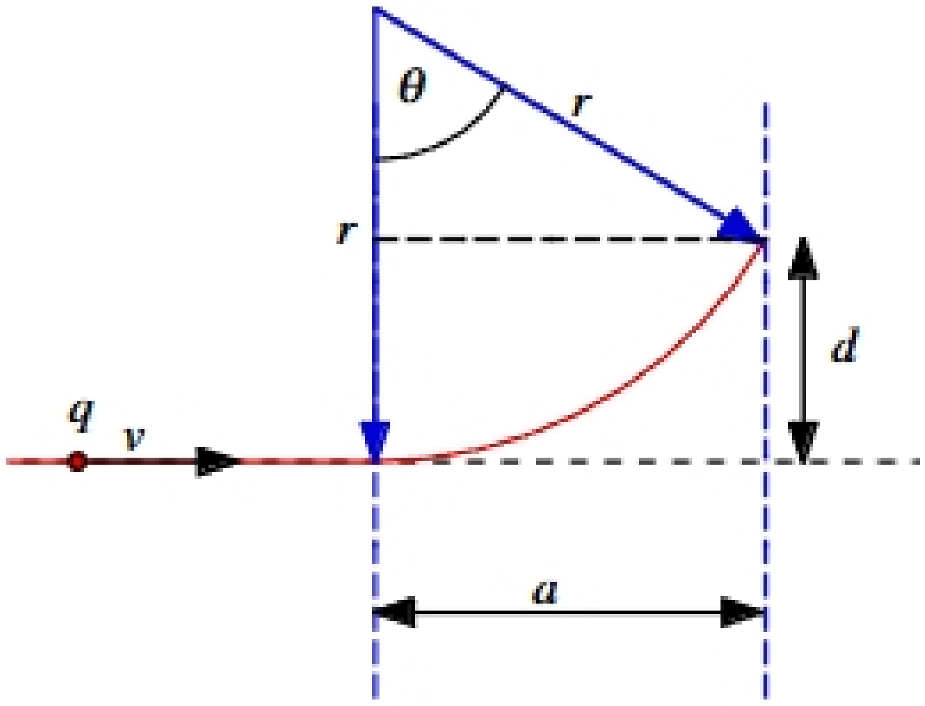

Figure 2. Problem 5.2.

The electric current in a wire is due to the motion of the electrons in the wire. The direction of current is defined to be the direction in which the positive charges move. Therefore, in a conductor the current is directed opposite to the direction of the electrons. The magnitude of the current is defined as the total charge per unit time passing a given point of the wire ($I = dq/dt$). If the current flows in a region with a non-zero magnetic field then each electron will experience a magnetic force. Consider a tiny segment of the wire of length dl. Assume that the electron density is $-\lambda$ C/m and that each electron is moving with a velocity v. The magnetic force exerted by the magnetic field on a single electron is equal to

$$d\overline{F}_{1e} = -e\left(\overline{v} \times \overline{B}\right)$$

A segment of the wire of length dl contains $\lambda\, dl/e$ electrons. Therefore the magnetic force acting in this segment is equal to

$$d\overline{F}_{magnetic} = \frac{\lambda dl}{e} d\overline{F}_{1e} = -\lambda dl\left(\overline{v} \times \overline{B}\right) = \lambda v\left(d\overline{l} \times \overline{B}\right) = I\left(d\overline{l} \times \overline{B}\right)$$

Here we have used the definition of the current I in terms of dq and dt: