Managerial Economics

Math Review

Work to Understand the Principles

- You can pass a history class by simply memorizing a set of dates, names and events. You will find, however, that in order to pass a math class you will need to do more than just memorize a set of formulas.
- While there is certainly a fair amount of memorization of formulas in a math class you need to do more. You need to understand how to USE the formulas and that is often far different from just memorizing them.
- Some formulas have restrictions on them that you need to know in order to correctly use them. For instance, in order to use the quadratic formula you must have the quadratic in standard form first. You need to remember this or you will often get the wrong answer!

- Other formulas are very general and require you to identify the parts in the problem that correspond to parts in the formula.
- If you don't understand how the formula works and the principle behind it, it can often be very difficult to use the formula.
- For example, in a calculus it's not terribly difficult to memorize the formula for differentiation. However, if you don't understand how to actually use the formula , you will find the memorized formula worthless.

Mathematics is Cumulative

- You've always got to remember that mathematics courses are cumulative. Almost everything you do in a math class will depend on subjects that you've previously learned. This goes beyond just knowing the previous sections in your current class to needing to remember material from previous classes.
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- You will find a college algebra class to be very difficult without the knowledge that you learned in your high school algebra class. You can't do a calculus class without first taking (and understanding) an Algebra and a Trigonometry class.

Review : Common Graphs

- The purpose of this section is to make sure that you're familiar with the graphs of many of the basic functions that you're liable to run across in class.

Example 1 Graph $y=-\frac{2}{5}x+3$

- *Solution*
- This is a line in the slope intercept form

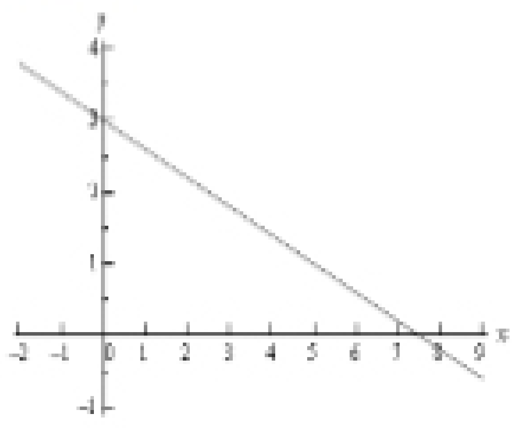
$$y=mx+b$$

- In this case the line has a *y* intercept of (0,*b*) and a slope of *m*. Recall that slope can be thought of as

$$m=\frac{\text{rise}}{\text{run}}$$

- Note that if the slope is negative we tend to think of the rise as a fall.
- The slope allows us to get a second point on the line. Once we have any point on the line and the slope we move right by *run* and up/down by *rise* depending on the sign. This will be a second point on the line.
- In this case we know (0,3) is a point on the line and the slope is . So starting at (0,3) we'll move 5 to the right (*i.e.*) and down 2 (*i.e.*) to get (5,1) as a second point on the line. Once we've got two points on a line all we need to do is plot the two points and connect them with a line.

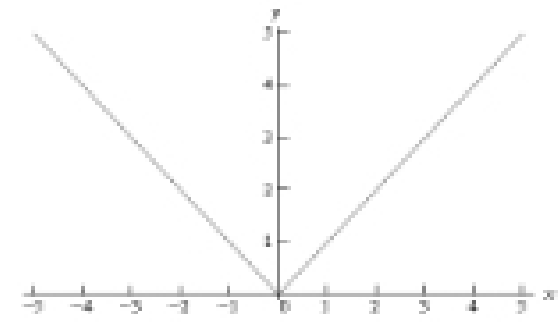
Here's the sketch for this line.



Example 2 Graph $f(x)=|x|$

Solution

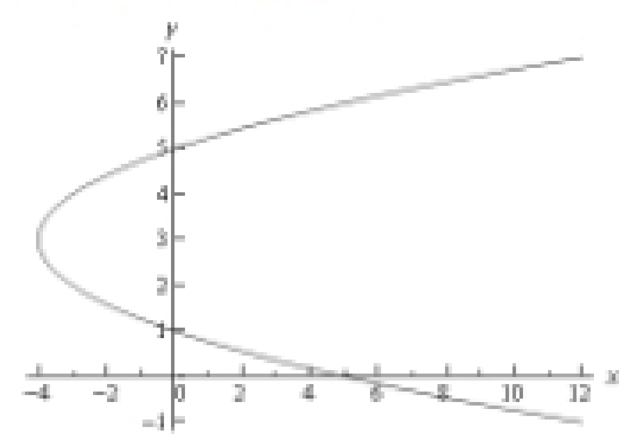
- There really isn't much to this problem outside of reminding ourselves of what absolute value is. Recall that the absolute value function is defined as,
- The graph is then, $|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$



Example 3 Graph $f(y)=y^2-6y+5$

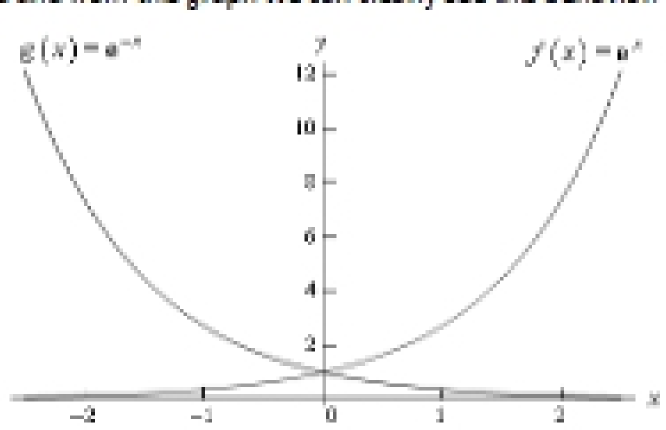
- *Solution*
- Most people come out of an Algebra class capable of dealing with functions in the form y=f(x) . However, many functions that you will have to deal with in a Calculus class are in the form x=f(y) and can only be easily worked with in that form. So, you need to get used to working with functions in this form.
- The nice thing about these kinds of function is that if you can deal with functions in the form y=f(x) then you can deal with functions in the form x=f(y) even if you aren't that familiar with them.
- Let's first consider the equation $y=x^2-6x+5$
- This is a parabola that opens up and has a vertex of (3,-4)

- Here's a sketch of the graph.



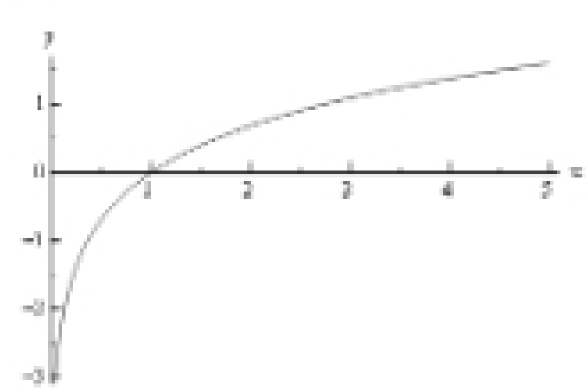
Example 4: Graph $f(x)=e^x$ and $g(x)=e^{-x}$

- These will both show up with some regularity in later sections and their behavior as *x* goes to both plus and minus infinity will be needed and from this graph we can clearly see this behavior.



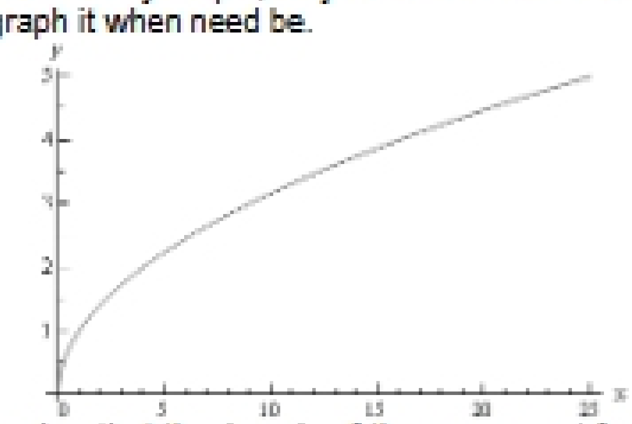
Example 5 Graph $f(x)=\ln(x)$

- ***Solution***



Example 6 Graph $y=\sqrt{x}$

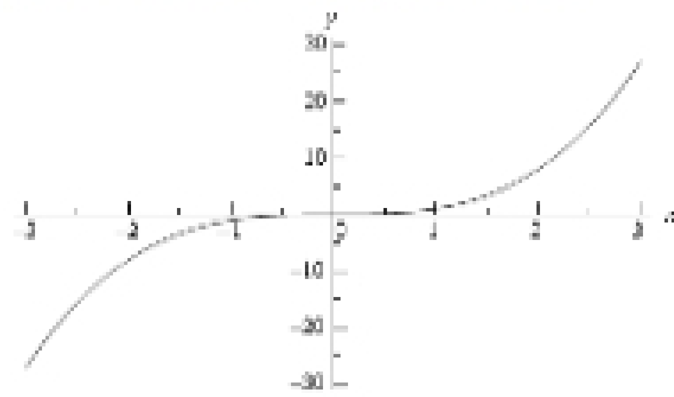
- *Solution*
- This one is fairly simple, we just need to make sure that we can graph it when need be.



- Remember that the domain of the square root function is . $x \geq 0$

Example 7 Graph $y=x^3$

- *Solution*
- Again, there really isn't much to this other than to make sure it's been graphed somewhere so we can say we've done it.



Interpretations of the Derivative

- All of these interpretations arise from recalling how our definition of the derivative came about.
- Rate of Change
- The first interpretation of a derivative is rate of change. This is the most important interpretation of the derivative. If f(x) represents a quantity at any *x* then the derivative f'(a) represents the instantaneous rate of change of f(x) at x=a.

Example 1

- Suppose that the amount of water in a holding tank at *t* minutes is given by $V(t)=2t^2-16t+35$
- Determine each of the following.
- (a) Is the volume of water in the tank increasing or decreasing at t=1 minute?
- (b) Is the volume of water in the tank increasing or decreasing at t=5 minutes?
- (c) Is the volume of water in the tank changing faster at t=1 or t=5 minutes?
- (d) Is the volume of water in the tank ever not changing? If so, when?

- *Solution*
- We are going to need the rate of change of the volume to answer these questions. This means that we will need the derivative of this function since that will give us a formula for the rate of change at any time *t*.
- The derivative is. $V'(t)=4t-16$ OR $\frac{dV}{dt}=4t-16$
- If the rate of change was positive then the quantity was increasing and if the rate of change was negative then the quantity was decreasing.

- **(a) Is the volume of water in the tank increasing or decreasing at t=1 minute?**
- In this case all that we need is the rate of change of the volume at t=1 or,

$$V'(1)=-12 \quad \text{OR} \quad \left.\frac{dV}{dt}\right|_{t=1}=-12$$

- So, at t=1 the rate of change is negative and so the volume must be decreasing at this time.