MA 15200   Lesson 27   Section 4.1

We have discussed powers where the exponents are integers or rational numbers. There also exists powers such as $2^{\sqrt{3}}$. You can approximate powers on your calculator using the power key. On most one-liner scientific calculators, the power key looks like $\boxed{y^x}$

Enter the base into the calculator first, press the power key, enter the exponent, and press enter or equal.

Ex 1: Approximate the following powers to 4 decimal places.

a) $2^{3\sqrt{2}} =$

b) $(2.3)^{4.8} =$

I Exponential Functions

An **exponential function *f* with base *b*** is defined by $f(x) = b^x$ or $y = b^x$, where b is a positive constant other than 1 and x is any real number.

> The following are **not** exponential functions. Why?
> $f(x) = x^3$ $f(x) = 1^x$
> $f(x) = (-4)^x$ $f(x) = x^x$

A calculator may be needed to evaluate some function values of exponential functions. (See example 1 above.)

Many real life situations model exponential functions. One example given in your textbook models the average amount spent (to the nearest dollar) at a shopping mall after x hours and is $f(x) = 42.2(1.56)^x$. The base of this function is 1.56. Notice there is also a 'constant' (42.2) multiplied by the power. Be sure to follow the order of operations; find the exponent power first, then multiply that answer by the 42.2.

Suppose you wanted to find the amount spent in a mall after browsing for 3 hours. Let x = 3.

$$f(3) = 42.2(1.56)^3$$
$$= 42.2(3.796416)$$
$$= 160.2087552$$

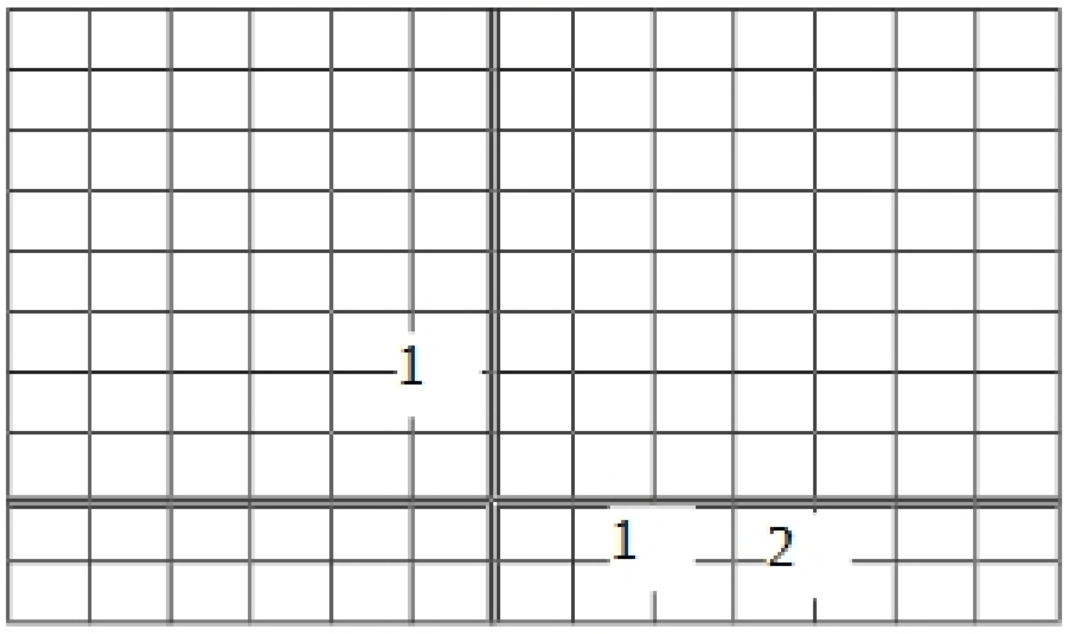
To the nearest dollar, a person on average would spend $160.

II Graphing Exponential Functions

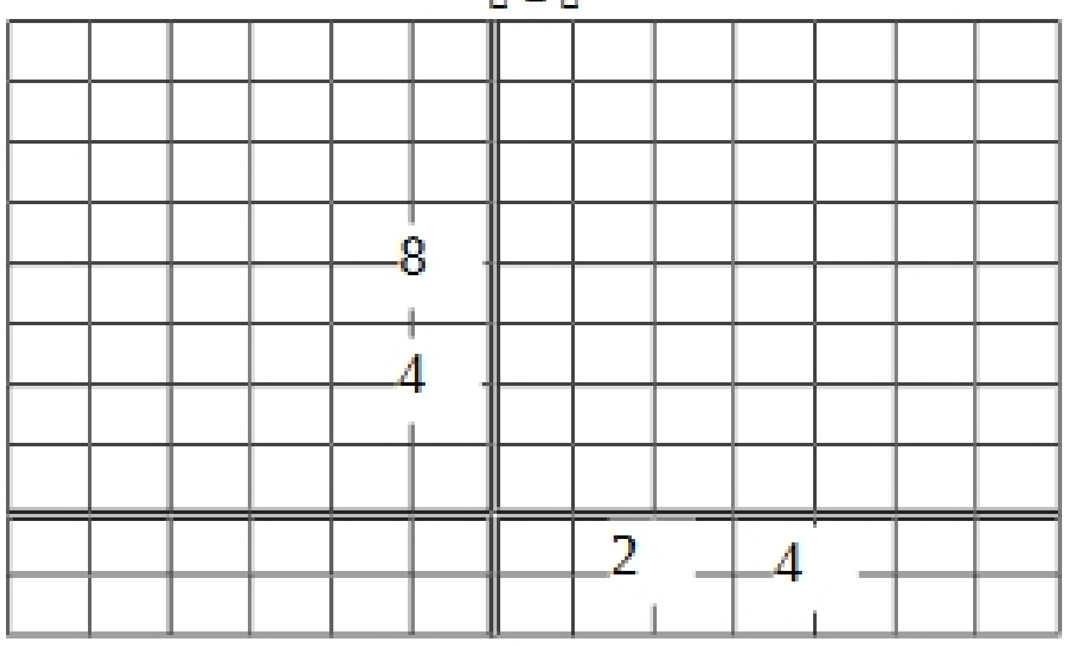
> To graph an exponential function, make a table of ordered pairs as you have for other types of graphs. Notice: If x = 0 for b^x, the value is 1 (zero power is 1). For a basic exponential function, the y-intercept is 1.
>
> Also, notice that y values will always be positive, so the graph always lies above the x-axis.

Ex 2: Graph each exponential function.

a) $y = \left(\frac{3}{2}\right)^x$



b) $f(x) = \left(\frac{1}{3}\right)^x$



There are several exponential graphs shown in figure 4.4 on page 415 of the text. After examining several graphs, the following characteristics can be found.

Characteristics of Exponential Functions of the form ***f(x)*** **=** b^x

1. The domain of the function is all real numbers $(-\infty, \infty)$ and the range is all positive real numbers $(0, \infty)$ (graph always lies above the *x*-axis).
2. Such a graph will always pass through the point (0, 1) and the *y*-intercept is 1. There will be no *x*-intercept.
3. If the base *b* is greater than 1 $(b > 1)$, the graph goes up to the right and is an increasing function. The greater the value of *b*, the steeper the increase (exponential growth).
4. If the base is between 0 and 1 $(0 < x < 1)$, the graph goes down to the right and is a decreasing function (exponential decay). The smaller the value of *b*, the steeper the decrease.
5. The graph represents a 1-1 function and therefore will have an inverse.
6. The graph approaches but does not touch the *x*-axis. The *x*-axis is known as an asymptote.

III The Natural Base *e* and the Natural Exponential Function

There is an irrational number, whose symbol is *e*, that is used quite often for exponential function's base. This number is the value of $\left(1+\frac{1}{n}\right)^n$ as *n* becomes very, very large or goes to infinity. An approximation of this number is $e = 2.718281827$ and the number *e* is called the **natural base**. The function $f(x) = e^x$ is called the **natural exponential function.** To approximate the powers of *e*, use these steps on your calculator.

1. Enter the exponent in your calculator.
2. Because the *e* power is above the *ln* key, you must press the [2nd] key first and then the [ln] key.
3. The value is approximately the power.

<u>Ex 3:</u> Approximate each power to 4 decimal places.

a) $e^3 =$

b) $e^{0.024} =$

c) $e^{-\frac{2}{3}} =$

Another life model that uses an exponential function is $f(x) = 1.26e^{0.247x}$, which approximates the gray wolf population of the Northern Rocky Mountains *x* years after 1978. (Notice: Multiply 0.247 by *x*, find the number *e* to that power, then multiply the result by 1.26.)

<u>Ex 4</u>: Use the model above the approximate the gray wolf population in 2008.
2008 is 30 years after 1978. Let *x* = 30.

$$f(30) = 1.26e^{0.247(30)}$$
$$= 1.26e^{7.41}$$
$$= 1.26(1652.426347)$$
$$\approx 2082$$

2082 gray wolves

IV Compound Interest

One of the most common models of exponential functions used in life are the models of compound interest. You know that the Simple Interest Formula is $I = Prt$ and the amount accumulated with simple interest is $A = P + Prt$. However, in this model, interest is only figured at the very end of the time period. In most situations, interest is determined more often; sometimes annually, monthly quarterly, etc. Then the amount accumulated becomes the formula on the next page.

Compound Interest Formula:
If an account has interest compounded *n* times per year for *t* years with principal *P* and an annual interest rate *r* (in decimal form), the amount of money in the account is found by