

1.

(30 pts.) Find the derivatives of the following functions (you need not simplify):

$$(a) f(x) = \frac{\ln(\sqrt{x})}{e^{(3-x)}} - \frac{3}{\ln(x)}$$

$$(b) f(x) = \sqrt[6]{\ln(x^4 + 4)} - e^{\pi}$$

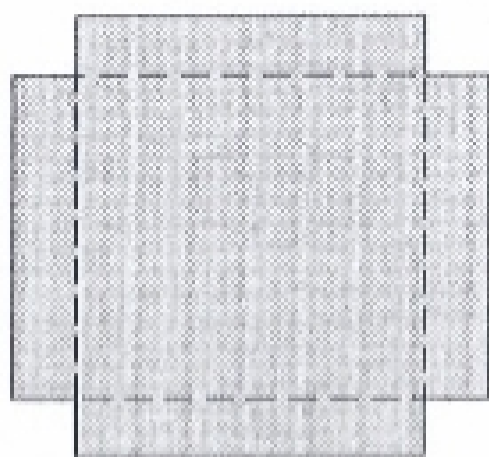
$$(c) f(x) = x^2 \ln(x^3) - e^{(e^{-x} + 3x)}$$

2. (5 pts.) The population of a certain country t years from now is predicted to be $P(t) = 2 e^{\left(\frac{t}{100}\right)}$ million people. Find its instantaneous and relative rates of change at this time ($t=0$).

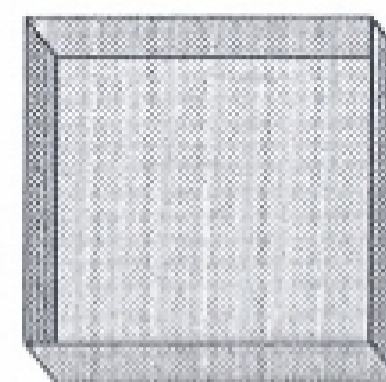
3. (15 pts.) An open-top box is to be made from a square sheet of cardboard 12 inches on each side by cutting a square from each corner and folding up the sides as shown in the



Square sheet



Corners removed



Side flaps folded up to make open-top box.

diagrams

below. Find the volume of the largest box that can be made this way.

4. (20 pts.)

$$f(x) = -2x^3 + 3x^2 + 12x; f'(x) = -6x^2 + 6x + 12 = -6(x - 2)(x + 1); \text{ and } f''(x) = -12x + 6 = -6(2x - 1).$$

(a) Find the critical numbers and the inflection points of f . (b) Construct sign charts for the first and second derivatives. (c) Find all open intervals of increase and decrease and open intervals on which the graph is concave up and concave down. (d) Classify each critical point as a relative maximum, relative minimum or neither. (e) Sketch the graph of $y = f(x)$ by hand, using and labeling **only** the relative extreme points, the inflection points and the y-intercept.

Note that $f(-2) = 4$, $f(-1) = -7$, $f(-\frac{1}{2}) = -5$, $f(\frac{1}{2}) = 6.5$, $f(1) = 13$, and $f(2) = 20$.