

NAME: Solution

ID NUMBER \_\_\_\_\_

**PART I** consists of 16 multiple choice questions (worth 5 points each) for a total of 80 points. Mark the correct answer on the answer card. For part I only the answer on the card will be graded.

**Part II** consists of 2 handgraded problems (worth 10 points each). A correct answer without supporting work may get no credit. Present a readable, orderly sequence of steps showing how you got your answer.

Part I ( 80 points ) :

1) Find  $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x^3}$  .  $\left( \frac{0}{0} \right)$ .

A) 0 **B)  $\frac{1}{6}$**  C)  $\frac{1}{4}$  D)  $\frac{1}{3}$  E)  $\frac{1}{2}$  F) 1 G)  $\frac{5}{6}$  H)  $\frac{4}{5}$  I)  $\frac{2}{3}$  J)  $\infty$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{3x^2} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\sin(x)}{6x} \left( \frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 0} \frac{\cos(x)}{6} = \boxed{\frac{1}{6}}$$

2) If  $f(x) = \frac{1}{\sqrt[3]{1-2x}}$  then find  $f'(0)$ .

A) 0 B)  $\frac{1}{6}$  C)  $-\frac{1}{4}$  D)  $\frac{1}{3}$  E)  $-\frac{1}{2}$  **F)  $\frac{2}{3}$**  G)  $-\frac{3}{2}$  H)  $\frac{4}{3}$  I)  $-\frac{2}{3}$  J) 1

$$f(x) = (1 - 2x)^{-1/3}$$

$$f'(x) = -\frac{1}{3} (1 - 2x)^{-4/3} (-2)$$

$$f'(x) = \frac{2}{3(1-2x)^{4/3}}$$

$$\boxed{f'(0) = \frac{2}{3}}$$

3) Find the y-coordinate of the absolute maximum point for the curve  $f(x) = \sin(x) + \cos(x)$  on the closed interval  $[0, \frac{\pi}{2}]$ .

- A)  $\sqrt{2}$  B)  $\sqrt{3}$  C)  $\frac{\sqrt{3}}{2}$  D)  $2\sqrt{3}$  E)  $3\sqrt{2}$  F)  $\frac{\sqrt{2}}{3}$  G)  $\frac{1}{2}$  H)  $\frac{3}{4}$  I)  $\frac{2}{3}$  J) 1

$$f'(x) = \cos(x) - \sin(x) = 0 \quad \text{when} \quad x = \frac{\pi}{4}$$

$f(0) = 1$  ,  $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$  (abs MAX)

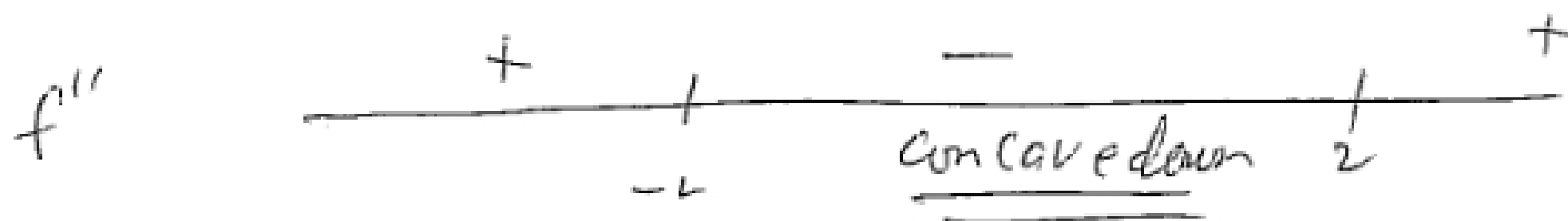
and  $f(\frac{\pi}{2}) = 1$

4) On which of the following intervals is the function  $f(x) = \frac{1}{12}x^4 - 2x^2 + 4$  always concave down ?

- A)  $(0, 4)$  B)  $(-\infty, 0)$  C)  $(4, \infty)$  D)  $(-4, 4)$  E)  $(-4, 0)$  F)  $(-\infty, -2)$   
 G)  $(2, \infty)$  H)  $(-2, 2)$  I)  $(-2, \infty)$  J) always concave up

$$f'(x) = \frac{1}{3}x^3 - 4x$$

$$f''(x) = x^2 - 4 = 0 \quad \text{for} \quad x = -2, 2$$



5) For the curve  $x^3 + y^3 - 9xy = 0$ , find  $\frac{dy}{dx}$ .

A)  $\frac{4x + 2y^2}{x^2 - 4y}$

B)  $\frac{4y + 2x^2}{y^2 - 4x}$

C)  $\frac{4x - 2y^2}{x^2 + 4y}$

D)  $\frac{3x + 2y^2}{x^2 - 3y}$

E)  $\frac{3y + 2x^2}{x^2 - 3y}$

F)  $\frac{x + 3y^2}{3y^2 - x}$

G)  $\frac{3y - x^2}{y^2 - 3x}$

H)  $\frac{4x - 2y^2}{x^2 - 4y}$

I)  $\frac{3x - 3y^2}{x^2 - 3y}$

J)  $\frac{2y - 4x^2}{4y^2 - 2x}$

$$3x^2 + 3y^2 \frac{dy}{dx} - 9y - 9x \frac{dy}{dx} = 0$$

$$(3y^2 - 9x) \frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y - 9x} = \boxed{\frac{3y - x^2}{y - 3x}}$$

6) Find all the critical values (x-coordinates) of  $f(x) = x^{\frac{2}{3}}(x+2)$ .

A) -2, 0

B)  $\frac{1}{2}, 1$

C)  $-\frac{2}{3}, \frac{2}{3}$

D) -1, 1

E)  $0, \frac{3}{4}$

F) -1, 0

G) 1, 2

H)  $-\frac{4}{5}, 0$

I)  $1, \frac{4}{5}$

J)  $0, \frac{3}{5}$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} (x+2) + x^{\frac{2}{3}}$$

$$= \frac{2(x+2)}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}}$$

$$= \frac{2x + 4 + 3x}{3x^{\frac{1}{3}}}$$

$$= \frac{5x + 4}{3x^{\frac{1}{3}}}$$

$$\boxed{\begin{array}{l} f' = 0 \quad \text{for } x = -\frac{4}{5} \\ f' \text{ DNE} \quad \text{for } x = 0 \end{array}}$$