Math 132

Fall 2006 Exam I

1. A Riemann sum $\sum_{j=1}^{N} f(\xi_j)\,\Delta x$ for a function f on an interval $[a,b]$ is said to be a *lower Riemann sum* if, for each j, the point ξ_j in the j^{th} subinterval is chosen so that $f(\xi_j)$ is minimized. Calculate the lower Riemann sum for $f(x) = x^2$, $[a,b] = \left[-\frac{3}{2}, \frac{5}{2}\right]$, and $N = 4$.

(Use a partition of [a,b] into equal length subintervals.)

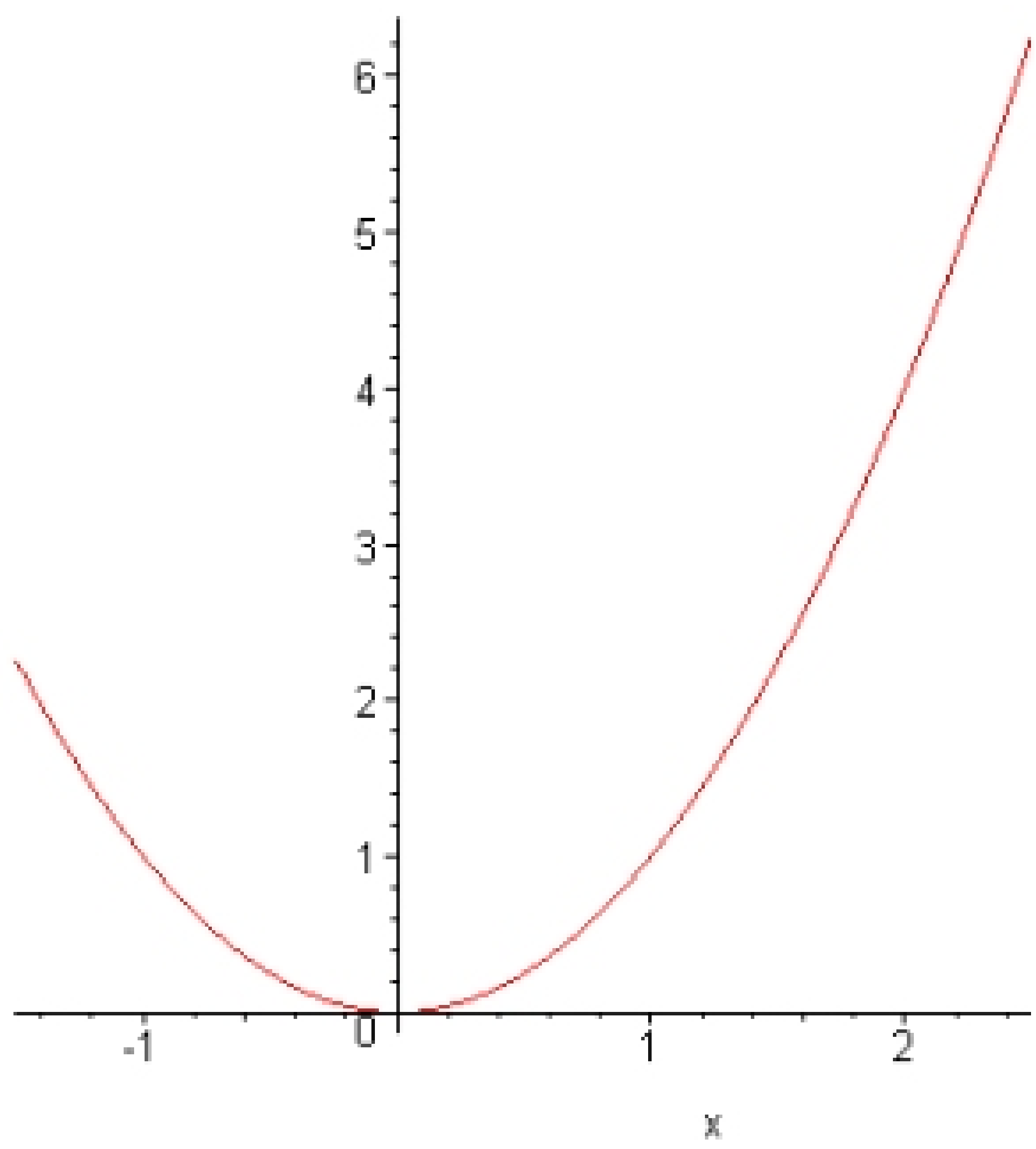
a) 7/42 b) 2 c) 9/4 d) 5/2 e) 11/4
f) 3 g) 13/4 h) 7/2 i) 15/4 j) 4

Solution : (e)

```
> f := x -> x^2;
```

$$f := x \rightarrow x^2$$

```
> plot(f(x),x=-3/2..5/2);
```



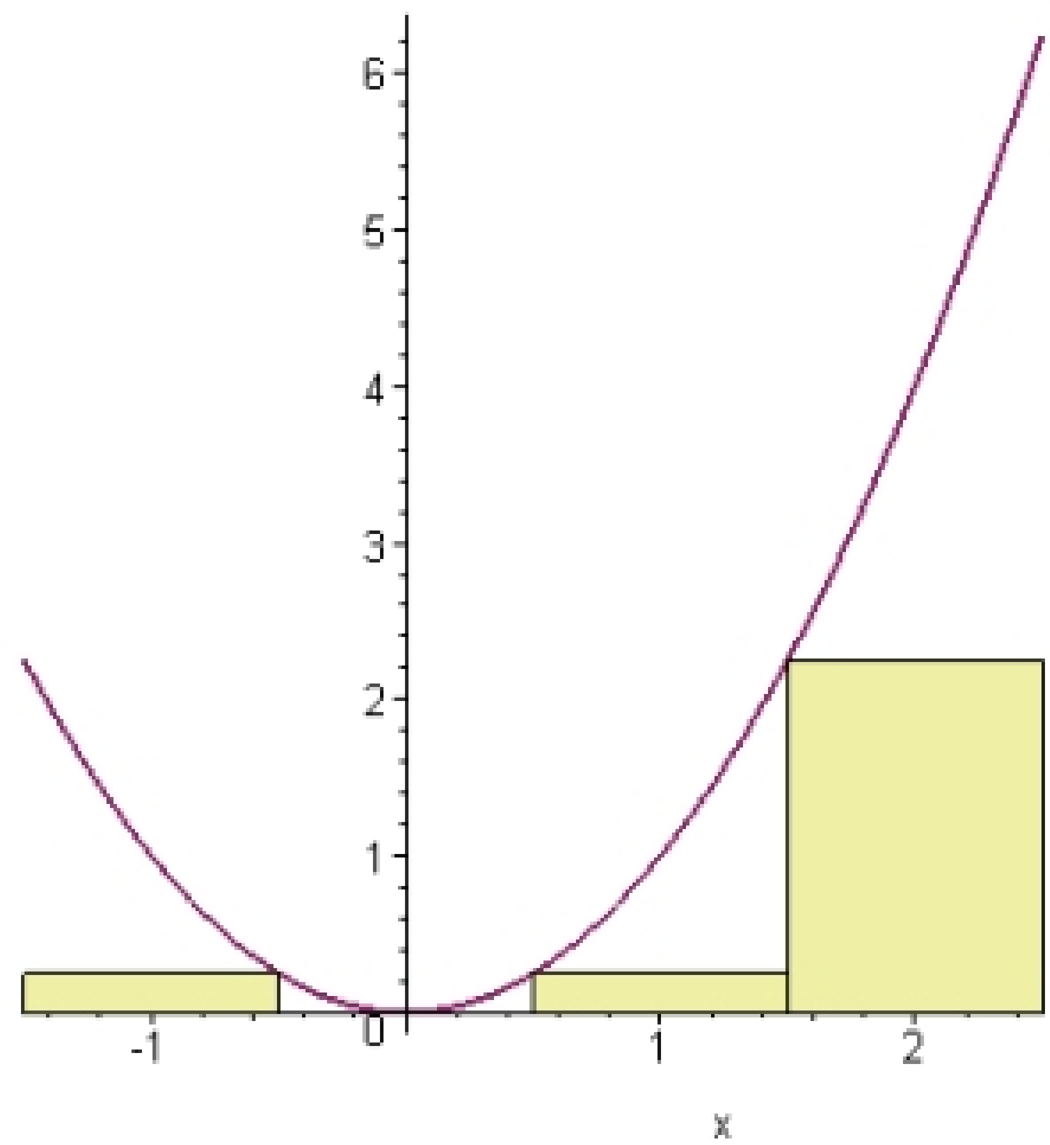
```
> a := -3/2:  b := 5/2:  N := 4:

> Delta :=  (b-a)/N;
```

$$\Delta := 1$$

```
> with(plottools): with(plots):
```

```
Warning, the names arrow and changecoords have been redefined
```

```
> r1 := rectangle([-3/2,f(-1/2)], [-1/2,0], color=COLOR(RGB, 0.94,0.94,0.65)):
  r2 := rectangle([1/2,f(1/2)], [3/2,0], color=COLOR(RGB, 0.94,0.94,0.65)):
  r3 := rectangle([3/2,f(3/2)], [5/2,0], color=COLOR(RGB, 0.94,0.94,0.65)): fnPlot := plot(f(x),x=-3/2..5/2,
  thickness=2,color=MAROON):
  display(r1,r2,r3,fnPlot);
```




```
> (f(-1/2)+f(0)+f(1/2)+f(3/2))*Delta;
```

$$\frac{11}{4}$$

2. Calculate $\int_0^{\frac{\pi}{3}} \sec(\theta)^2 \, d\theta$.

a) 1 **b)** $\sqrt{2}$ **c)** $\sqrt{3}$ **d)** 2 **e)** $\frac{3\sqrt{2}}{2}$

f) $\frac{3\sqrt{3}}{2}$ **g)** $2\sqrt{2}$ **h)** $2\sqrt{3}$ **i)** $3\sqrt{2}$ **j)** $3\sqrt{3}$

Solution : (c)

```
> int(sec(theta)^2,theta = 0 .. Pi/3);
```

$$\sqrt{3}$$

```
> J := Int(sec(theta)^2,theta);
  antiderivative := value(J);
  definiteIntegral :=
    subs(theta=Pi/3, antiderivative) - subs(theta=0, antiderivative);
```

$$J := \int \sec(\theta)^2 \, d\theta$$

$$antiderivative := \frac{\sin(\theta)}{\cos(\theta)}$$

$$definiteIntegral := \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)} - \frac{\sin(0)}{\cos(0)}$$

```
> simplify( definiteIntegral );
```

$$\sqrt{3}$$

3. An antiderivative of f **is the function** $x \to \frac{3x^2 + 4x + 2}{x^2 + x + 1}$ **. If** $\int_0^b f(x) \, dx = 1$ **, then**