

- 20 multiple choice questions worth 5 points each.
- Exam is comprehensive

- No calculators!
- For the multiple choice questions, mark your answer on the answer card.

### Useful Formulas

$\sum_{i=1}^n i = \frac{n(n+1)}{2}$	$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$	$\sin^2 \theta + \cos^2 \theta = 1$
$1 + \tan^2 \theta = \sec^2 \theta$	$1 + \cot^2 \theta = \csc^2 \theta$
$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$	$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \cos(A - B)]$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin(2\theta) = 2 \sin \theta \cos \theta$	$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$
$\int \csc x \, dx = -\ln  \csc x + \cot x  + C$	$\int \sec x \, dx = \ln  \sec x + \tan x  + C$

1. Find a power series representation for  $f(x) = \frac{2x}{(1-3x)^2}$  centered at 0.

A.  $\sum_{n=1}^{\infty} 2 \cdot 3^n (n+1)x^n$

B.  $\sum_{n=1}^{\infty} -3^n nx^{n-1}$

C.  $\sum_{n=1}^{\infty} 2 \cdot 3^n (n-1)x^n$

D.  $\sum_{n=1}^{\infty} 2 \cdot 3^{n-1} nx^n$

E.  $\sum_{n=1}^{\infty} 3^{n+1} nx^{n-1}$

F.  $\sum_{n=1}^{\infty} -2 \cdot 3^{n-1} x^{n+1}$

G.  $\sum_{n=1}^{\infty} 2 \cdot 3^n x^{2n}$

2. Find the third degree Taylor polynomial,  $T_3$ , centered at  $x = 2$  for the function  $f(x) = \frac{1}{1+x}$ .

A.  $f(x) = 1 - x + x^2 - x^3$

B.  $f(x) = \frac{1}{3} - \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2 - \frac{1}{81}(x-2)^3$

C.  $f(x) = 1 - (x-2) + (x-2)^2 - (x-2)^3$

D.  $f(x) = \frac{1}{3} + \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2 + \frac{1}{81}(x-2)^3$

E.  $f(x) = 1 + x + x^2 + x^3$

F.  $f(x) = \frac{1}{3} - \frac{1}{9}(x-2) + \frac{2}{27}(x-2)^2 - \frac{6}{81}(x-2)^3$

G.  $f(x) = \frac{1}{3} - \frac{1}{9}x + \frac{1}{27}x^2 - \frac{1}{81}x^3$

3. Find the Taylor series for  $f(x) = e^{3x}$  centered at 1.

A.  $\sum_{n=0}^{\infty} \frac{3^n e^3 (x-1)^n}{n!}$

B.  $\sum_{n=0}^{\infty} \frac{3^n (x-1)^n}{n!}$

C.  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

D.  $\sum_{n=0}^{\infty} \frac{3^n e^3 x^n}{n!}$

E.  $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(3n)!}$

F.  $\sum_{n=0}^{\infty} \frac{3e(x+1)^n}{n!}$

G.  $\sum_{n=0}^{\infty} \frac{3^{n+1} e^3 (x-1)^n}{n!}$

4. Find the value of the series or conclude that it diverges:

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{2}\right)^{2n+2}}{(2n+1)!} = \frac{\left(\frac{\pi}{2}\right)^2}{1!} - \frac{\left(\frac{\pi}{2}\right)^4}{3!} + \frac{\left(\frac{\pi}{2}\right)^6}{5!} - \frac{\left(\frac{\pi}{2}\right)^8}{7!} + \dots$$

A.  $\frac{\pi^2}{4}$

B.  $\frac{1}{\sqrt{2}}$

C.  $\frac{\sqrt{3}}{2}$

D.  $\frac{\pi}{2\sqrt{2}}$

E.  $\frac{\pi^2}{4}$

F.  $\frac{\pi}{2}$

G.  $\frac{\pi\sqrt{3}}{2}$

H. The series diverges.