

Week 9 Key Topics

8.4 The Binomial Distribution

Bernoulli or Binomial Experiment Properties

- The number of trials in the experiment is fixed.
- There are two outcomes of the experiment: success and failure.
- The probability of success in each trial is the same.
- The trials are independent of each other.

Computing Binomial Probabilities

- In a binomial experiment, in which the probability of success in any trial is p and the probability of failure in any trial is $q = 1 - p$, the probability of exactly r successes in n independent trials is $P(X = r) = C(n, r)p^r q^{n-r}$.
- X is called the *binomial random variable* and the probability distribution of X is the *binomial distribution*.

For a binomial probability question you must do the following:

- Decide that it is a Bernoulli trial by checking the 4 criteria.
- Define success.
- Find the number of times the experiment is performed, n .
- Find the probability of success, p .
- Determine the desired number of successes, r .

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p and probability of failure $q = 1 - p$, then

- mean $\mu = E(X) = np$
- variance $\text{Var}(X) = npq$
- standard deviation $\sigma_x = \sqrt{npq}$

8.5 The Normal Distribution

Continuous Probability Distribution

- Recall that a continuous random variable can take on any value.
- A *continuous probability distribution* is a probability distribution associated with a continuous random variable.
 - Defined by a function f whose domain is the interval of real values taken on by the random variable X associated with the experiment.

Probability Density Function (pdf)

- $f(x)$, the continuous probability distribution, is nonnegative for all values of x
- The area of the region between the graph of f and the x -axis is equal to 1.
- $P(a < X < b)$ is the probability that the random variable X assumes a value between a and b , ($a < x < b$).
 - Given by the area of the region between $f(x)$ and the x -axis from $x = a$ to $x = b$.

- o If you have had calculus before, this is $\int_a^b f(x)dx$.

Graphs of Probability Distributions

- Discrete finite variables: Graph the probability as a histogram.
 - Each rectangle has a base of width 1 (centered on the values of the r. v.) and a height equal to the probability of that value of the r. v.
 - The area, length \times height, is the probability that a given value of the r. v. occurs.
 - To find the probability of a range of X values, add up the areas over the range of X values.
- Continuous variables: Graph the probability distribution as a smooth curve.
 - The area above the range of X values is the probability of these X values occurring.
 - Many natural sciences, behavioral sciences, psychological test scores, and physical phenomena have a continuous distribution with a bell-shaped curve.
 - These special families of important continuous probability distributions are known as the *normal distributions*.
 - The *standard normal distribution* has mean $\mu = 0$ and $\sigma = 1$.
 - Use Z instead of X as the continuous random variable.

Properties of Normal Curves

- The curve has a peak at $x = \mu$ and is symmetric with respect to the vertical line $x = \mu$.
- The curve always lies above the x -axis but approaches the x -axis as x extends infinitely in either direction.
- The area under the entire curve is 1.
- 68.27% of the area under the curve lies within 1 standard deviation of the mean (that is, between $\mu - \sigma$ and $\mu + \sigma$).
- 95.45% of the area lies within 2 standard deviations of the mean.
- 99.73% of the area lies within 3 standard deviations of the mean.
- The shape is completely determined by μ and σ .

8.6 Applications of the Normal Distribution

The Normal Curve Approximation to the Binomial Distribution

- We will use the normal curve to approximate the binomial distribution.
- As the number of trials increases, the binomial distribution comes closer and closer to the normal curve approximation.
- If you are asked to approximate a given binomial probability with n trials, probability of success p , and probability of failure q , you will use a normal distribution with $\mu = np$ and $\sigma = npq$.
- To determine the boundaries for the normal curve approximation, identify the rectangles in the binomial histogram that should be included. Then adjust the boundaries by ± 0.5 to move to the left or right endpoint of a rectangle.