

If a box with a square cross section is to be sent by a delivery service, there are restrictions on its size such that its volume is given by $V = x^2(108 - 4x)$, where x is the length of each side of the cross section (in inches).

a) Is V a function of x ?

Yes, every x gives only one V

b) If $V = V(x)$, find $V(10)$ and $V(20)$.

$$\begin{aligned} V(10) &= (10)^2(108 - 40) \\ &= 100(108 - 40) \\ &= 6800 \text{ in}^3 \end{aligned}$$

$$\begin{aligned} V(20) &= 20^2(108 - 80) \\ &= 400(108 - 80) \\ &= 11,200 \text{ in}^3 \end{aligned}$$

c) What restrictions must be placed on x (the domain) so that the problem makes physical sense?

$$\begin{aligned} &\left\{ \begin{array}{l} x > 0 \\ x^2(108 - 4x) > 0 \\ x^2 > 0 \end{array} \right. \quad \begin{array}{l} 108 - 4x > 0 \\ x > 27 \\ 0 < x < 27 \text{ in} \end{array} \end{aligned}$$

Function Operations

Sum $(f+g)(x) = f(x) + g(x)$

Difference $(f-g)(x) = f(x) - g(x)$ ← distribute negative

Product $(fg)(x) = f(x) \cdot g(x)$

Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Composition $(f \circ g)(x) = f(g(x))$

For example: $f(x) = (x-1)^2$ and $g(x) = 1-2x$

Find the following:

$$\begin{aligned} (f+g)(x) &= (x-1)(x-1) + 1-2x \\ &= x^2 - 2x + 1 + 1 - 2x \\ &= x^2 - 4x + 2 \leftarrow \end{aligned}$$

$$\begin{aligned} (f-g)(x) &= x^2 - 2x + 1 - 1 + 2x \\ &= x^2 \leftarrow \end{aligned}$$