Section I For each of the problems in the this section at least one of the choices is correct. For some of the problems more than one of the choices is correct. Record your answers for problems in this section on the answer sheet (last page).

1. (6 pts) Consider the following matrix $A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ -2 & 1 & 3 & 3 \\ -1 & 2 & 1 & 2 \end{bmatrix}$. Which of the following matrices are row-equivalent to A.

a. $\begin{bmatrix} 1 & -2 & 0 & 3 \\ -2 & 1 & 3 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 1 & 2 \end{bmatrix}$ f. $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. (8 pts) Consider the following four matrices:

$$A = \begin{bmatrix} -3 & 0 & 1 \\ 1 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 7 & -1 \\ 3 & -4 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 \\ 2 & -3 \\ 1 & 1 \end{bmatrix}$$

Which of the following arithmetic operations are possible?

a. $3A - 2B$ b. AB c. $AC - D$ d. B^2

e. C^2 f. $(DA)^2$ g. D^2A^2 h. $DB - BD$

3. (8 pts) Consider the following subsets of $\mathbb{R}^3$:

a. $\{(x_1, x_2, x_3)^T \mid x_1 > 0 \text{ or } x_2 > 0\}$ b. $\{(x_1, x_2, x_3)^T \mid (x_1 = 0) \text{ or } (x_2 = 0) \text{ or } (x_3 = 0)\}$.

c. $\{(x_1, x_2, x_3)^T \mid x_1 - x_3 = 0\}$ d. $\{(x_1, x_2, x_3)^T \mid x_1^2 + x_2^2 + x_3^2 = 1\}$

Which of the above subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$?

4. (8 pts) Consider the following subsets of P_4:

a. $\{p \in P_4 \mid p(1) = 0\}$

b. $\{p \in P_4 \mid p(1) \geq p(0)\}$

c. $\{p \in P_4 \mid p \text{ is even}\}$

d. $\{p \in P_4 \mid \deg(p) = odd\}$

Which of the above subsets of P_4 are subspaces of P_4?

5. (8 pts) Consider the following subsets of $\mathbb{R}^3$:

a. $\left\{\begin{bmatrix}2\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\2\\3\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}1\\-2\\-2\end{bmatrix}, \begin{bmatrix}1\\5\\-3\end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix}-1\\0\\0\end{bmatrix}, \begin{bmatrix}5\\3\\-2\end{bmatrix}, \begin{bmatrix}-1\\-6\\4\end{bmatrix}\right\}$

d. $\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}3\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\-2\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\0\end{bmatrix}\right\}$

Which of the above subsets of $\mathbb{R}^3$ are spanning sets for $\mathbb{R}^3$?

6. (8 pts) Consider the following subsets of $\mathbb{R}^3$:

a. $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\2\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\-2\end{bmatrix}, \begin{bmatrix}-6\\-2\\4\end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix}0\\-1\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\-2\end{bmatrix}, \begin{bmatrix}1\\-2\\3\end{bmatrix}\right\}$

d. $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}-2\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\2\\1\end{bmatrix}, \begin{bmatrix}3\\1\\1\end{bmatrix}\right\}$

Which of the above subsets of $\mathbb{R}^3$ are linearly independent?

7. (4 pts) Consider the set $S = \{1+x, 1-x, 1-2x+x^2\}$ which is a subset of P_3. Is the set S linear independent?

8. (8 pts) Consider the following subsets of $\mathbb{R}^3$:

a. $\left\{\begin{bmatrix}-1\\-1\\0\end{bmatrix},\begin{bmatrix}-1\\-1\\1\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}0\\-2\\3\end{bmatrix},\begin{bmatrix}-2\\-2\\2\end{bmatrix},\begin{bmatrix}-5\\1\\-4\end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}-1\\-1\\3\end{bmatrix},\begin{bmatrix}-3\\2\\-6\end{bmatrix}\right\}$

d. $\left\{\begin{bmatrix}-1\\0\\1\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\1\end{bmatrix},\begin{bmatrix}2\\0\\1\end{bmatrix}\right\}$

Which of the above subsets of $\mathbb{R}^3$ forms a basis for $\mathbb{R}^3$?

9. (9 pts) Determine whether the following are linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^3$

a. $L(x)=\begin{bmatrix}x_1-4x_2+4x_4\\x_1x_2\\-2x_1+2x_2+x_3\end{bmatrix}$

b. $L(x)=\begin{bmatrix}x_4-1/x_2\\0\\x_4+1/x\end{bmatrix}$

c. $L(x)=\begin{bmatrix}x_1\\x_1-x_2\\x_1-x_2+x_3\end{bmatrix}$

10. (9 pts) Determine whether the following are linear transformations from P_3 to P_4

a. $L(p(x))=x^2p'(x)$

b. $L(p(x))=\int_0^x p(t)\,dt$

c. $L(p(x))=x(1+p(x))$

11. (6 pts) Consider the sets of vectors given below. Which sets are orthogonal sets?

a. $\left\{\begin{bmatrix}-1\\2\\-2\end{bmatrix},\begin{bmatrix}2\\-2\\-3\end{bmatrix},\begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix}-1\\2\\-3\\0\\1\end{bmatrix},\begin{bmatrix}2\\-1\\-3\\2\\-5\end{bmatrix}\right\}$

c. $\left\{[1,\ -2,\ -1,\ 0,\ 3,\ -1]^T,[12,\ 3,\ 4,\ -4,\ -2,\ -4]^T\right\}$