Note: the integral $\int \frac{x^3-1}{\sqrt{x}}$ should <u>not</u> be computed using supstitution

$$\int \frac{x^3-1}{\sqrt{x}}\,dx = \int \frac{x^3}{x^{1/2}} - \frac{1}{x^{1/2}}\,dx$$

$$= \int x^{5/2} - x^{-1/2}\,dx$$

$$= \frac{2}{7}x^{7/2} - 2x^{1/2} + C$$

$\boxed{5.2B}$ Solving Differential Equations by Seperation of variables

Example: Find the particular solution for the differential equation under the given condition

$$\frac{dy}{dx} = 4x^3y^2 \quad \text{where } y=2 \text{ when } x=1$$

(or $y(1)=2$)

$$\int y^{-2}\,dy = \int 4x^3\,dx$$

$$-y^{-1} = x^4 + C \rightarrow \frac{-1}{y} = x^4 + C$$

$$y = \frac{1}{-x^4+C} \longleftarrow \text{general solution}$$

$$y(1)=2 \quad 2 = \frac{1}{C-1} \rightarrow C = \frac{3}{2}$$

$$y = 2(3-2x^4)^{-1} \longleftarrow \text{particular solution}$$

Example: Solve $\frac{dy}{dx} = \frac{y^2+4}{xy}$ where $y=3$ when $x=1$

$$u = y^2+4 \rightarrow dy = \frac{du}{2y}$$

$$\int \frac{y}{y^2+4}\, dy = \int \frac{1}{x}\, dx$$

$$\frac{1}{2}\int \frac{1}{u}\, du = \int \frac{1}{x}\, dx$$

$$\frac{1}{2} \ln(y^2+4) = \ln(x) + C$$

$$\ln e^{(y^2+4)} = \ln(x^2) + C$$

$$y^2+4 = e^{\ln(x^2)} + C$$

$$y^2+4 = Cx^2$$

$y = \pm\sqrt{Cx^2-4} \leftarrow$ Two general solutions

$y(1) = 3 \quad 3 = \pm\sqrt{C(1)^2-4}$

$$C = 13$$

$y = \pm\sqrt{13x^2-4} \leftarrow$ Two particular Solutions

Example: Solve $\frac{dy}{dx} - xe^{-4y} = 2e^{-4y}$ where $y(0) = \frac{1}{2}$

$$\frac{dy}{dx} = 2e^{-4y} + xe^{-4y}$$

$$\frac{dy}{dx} = (2+x)e^{-4y}$$

$$\int e^{-4y}\,dy = \int (2+x)\,dx$$

$$\frac{1}{4}e^{4y} = \frac{1}{2}x^2 + 2x + C$$

$$e^{4y} = 2x^2 + 8x + C$$

$$4y = \ln|2x^2 + 8x + C|$$

$$y = \frac{1}{4}\ln(2x^2 + 8x + C) \leftarrow \text{general solution}$$

$$\frac{1}{2} = \frac{1}{4}\ln(C) \rightarrow 2 = \ln(C)$$

$$C = e^2$$

$$\therefore y = \frac{1}{4}\ln(2x^2 + 8x + e^2)$$