

Name _____

Score _____

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

Directions: If the technique you apply to solve a specific problem is to treat the problem as:

- a. a linear differential equation, then specifically identify the component pieces you construct as you solve the equation, i.e., specifically, the integrating factor
- b. an exact equation, then specifically verify that the equation is exact before proceeding

1. (10 pts) Classify each of the following differential equations by identifying their order and by identifying whether they are linear.

a. $x^2 \frac{d^3 y}{dx^3} - (x+1) \left(\frac{dy}{dx} \right)^2 - y = 0$

b. $(x^2 - 1)u'' - (x+1)u' + xu = \sin x$

c. $x^2 y' - 2xy = \sin(xy)$

d. $\frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = \cos(t + y)$

e. $\ddot{x} - \left(1 - \frac{\dot{x}}{2}\right)x = x + 1$

2. (10 pts) Find the general solution of the differential equation $y' + xy^2 + 2y^2 = 0$

3. (10 pts) Find the general solution of the differential equation $\frac{dy}{dx} = 2y + x + 5$

4. (10 pts) Find the general solution of the differential equation $\left(1 + \sin x - \frac{y}{x^2}\right)dx = \left(1 + y^2 - \frac{1}{x}\right)dy$

5. (10 pts) Find the general solution of the differential equation $(y^2 + 2yx)dx + x^2 dy = 0$

6. (10 pts) Solve the initial-value problem $(x^2 + 1)\frac{dy}{dx} = x + xy^2$, $y(0) = 2$

7. (10 pts) Solve the initial-value problem $xy' + y = xe^{-x^2}$, $y(1) = 2$

8. (10 pts) Solve the initial-value problem $(4y + 2x - 5)dx + (6y + 4x - 1)dy = 0$, $y(-1) = 2$

9. (10 pts) Find the general solution of the differential equation $\frac{dy}{dx} = y(xy^3 + 2)$

10. (14 pts) Separate the linear modeling problems (page 2) and take it with you as you leave class.

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Work carefully. Show all relevant supporting steps. **Do your own work** – Submitting solutions for this problem as a representation of your own work which, in fact, are the cumulative/collaborative efforts of others from this class and/or of others from the Missouri Club, etc. will result in a failing grade for the entire exam. (See OP 34.12).

- Part A-1. (5 pts) Initially, 120 milligrams of a radioactive substance was present. After 5 hours, the mass decreased by 2%. If the rate of decay is proportional to the amount of the substance present at time t , find the amount remaining after 25 hours.
- Part B-1 (9 pts) A thermometer is removed from a room where the temperature is 80°F and is taken outside, where the air temperature is 10°F . After one-half minute the thermometer reads 55°F . How long will it take for the thermometer to reach 15°F ?