

Name _____

Score _____

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

Part I. (18 pts) Do three (3) of the following four problems:

1. Find the Laplace transform of $f(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 2 & 1 \leq t < 4 \\ 0 & 4 \leq t < \infty \end{cases}$
2. Find the Laplace transform of $f(t) = (e^t + e^{-t})^2$
3. Find the Laplace transform of $f(t) = e^{3t}(t^2 + t)^2$
4. Find the Laplace transform of $f(t) = (t+5)u(t-2)$

Part II. (18 pts) Do three (3) of the following four problems:

5. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\left(\frac{2}{s} - \frac{3}{s^2}\right)^2\right)$
6. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{7s-5}{s^2+16}\right)$
7. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{2s+5}{s^2+6s+34}\right)$
8. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s-1)}\right\}$

Part III. (65 pts) Do five (5) of the following six problems:

9. Use the method of Laplace transforms to solve the following linear differential equation

$$y' - y = 2t$$

$$y(0) = -1$$

10. Use the method of Laplace transforms to solve the following linear differential equation

$$y'' - 5y' + 4y = 0$$

$$y(0) = 1, y'(0) = -1$$

11. Use the method of Laplace transforms to solve the following linear differential equation

$$y'' - 4y' + 4y = t$$

$$y(0) = 0, y'(0) = 1$$

12. Use the method of Laplace transforms to solve the following linear differential equation

$$y'' - 5y' + 6y = u(t-1)$$

$$y(0) = 0, y'(0) = 0$$

13. Use the method of Laplace transforms to solve the following linear differential equation

$$y'' + 4y = \sin t$$

$$y(0) = 0, y'(0) = 2$$

14. Use the method of Laplace transforms to solve the following linear differential equation

$$y'' + y = \delta(t - \pi)$$

$$y(0) = 1, y'(0) = -2$$