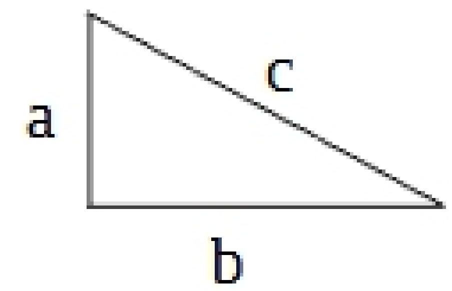
Math and Physics Refresher

This course assumes that you have studied Newtonian mechanics in a previous calculus-based physics course (i.e. PHY2060) and at least have co-registered in a vector calculus course (Calc 3). Listed below are some of the concepts in basic math, calculus, and physics that you are expected to know *or to acquire* during this course. This is not a complete summary of introductory math and physics. It is only meant to be a refresher of some of the concepts used in this course. Please report any inaccuracies to the professor.

Geometry



1. Pythagorean Theorem:
   The square of the hypotenuse of a right triangle is the sum of the squares of the two legs: $c^2 = a^2 + b^2$

2. Circumference of a circle: $C = 2\pi R$
3. Volume of a sphere: $V = \frac{4}{3}\pi R^3$
4. Surface area of sphere: $S = 4\pi R^2$

Calculus

1. Differentiation:

   You are expected to be able to take simple derivatives:
   $$\frac{d}{dx}x^n = nx^{n-1}$$
   $$\frac{d}{dx}\sin x = \cos x$$
   $$\frac{d}{dx}\cos x = -\sin x$$
   $$\frac{d}{dx}e^x = e^x$$

2. Product Rule

   $$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$$

   Example: $\frac{d}{dx}\left(x\sin x\right) = x\cos x + \sin x$

3. Chain Rule

$$\frac{d}{dx} f\big(g(x)\big) = \frac{df}{dg}\frac{dg}{dx}$$

Examples:

$$\frac{d}{dx}\sin 2x = 2\cos 2x$$

$$\frac{d}{dx}\exp\left(-x^2\right) = -2x\exp\left(-x^2\right)$$

4. Integration

You are expected to be able to perform simple integrals:

$$\int x^n dx = \frac{1}{n+1}x^{n+1}$$

$$\int dx\cos x = \sin x$$

$$\int dx\sin x = -\cos x$$

$$\int dx\, e^x = e^x$$

A purist would note that a constant should be added to these indefinite integrals.

5. Change of Variables

To use integration tables correctly, you must be able to change variables. For example:

$$I = \int_0^L \sin^2\left(\frac{n\pi x}{L}\right)$$

Let $u = \frac{n\pi x}{L} \quad \Rightarrow \quad dx = \frac{L}{n\pi}du$

$$I = \frac{L}{n\pi}\int_0^{n\pi} du\sin^2 u$$

Then use $\int dx\sin^2 x = \frac{x}{2} - \frac{1}{4}\sin 2x$

$$I = \frac{L}{n\pi}\frac{n\pi}{2} = \frac{L}{2}$$

6. Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Approximations

For small *x*, the following expansions are useful:

1. $\frac{1}{1-x} \approx 1 + x + x^2 + \cdots$
2. Binomial Expansion: $(1+x)^n \approx 1 + nx + \cdots$
3. Taylor Expansion: $f(x) \approx f(0) + x \left.\frac{df}{dx}\right|_{x=0} + \frac{x^2}{2!} \left.\frac{d^2 f}{dx^2}\right|_{x=0} + \cdots$

Differential Equations

We will study the solutions to several differential equations when we study circuits in this class. Although you are not required to have taken a course in differential equations, we will learn how to solve the simplest ones:

1. The exponential function is the only function whose derivative is the function itself:
   $$\frac{df}{dx} = \alpha f(x)$$
   $f(x) = Ce^{\alpha x}$ is the general solution, where C and α are constants

2. Two derivatives of the trigonometric functions give you back the same function with a sign change:
   $$\frac{d^2 f}{dx^2} = -k^2 f(x)$$
   $f(x) = A \sin kx + B \cos kx$ is the general solution, where A, B, and k are constants

   Using complex exponentials (see next section), you can also represent this solution as:
   $$f(x) = A' e^{ikx} + B' e^{-ikx}$$