

## Chapter 4

### Mathematical Induction

- Used to verify a property of a sequence
- $2, 4, 6, 8, \dots$  for  $i \geq 1$   $a_i = 2i$ 
  - infinite sequence with infinite distinct values
- for  $i \geq 1$   $b_i = (-1)^i$ 
  - infinite sequence with finite distinct values
- for  $1 \leq i \leq 6$   $c_i = i+5$ 
  - finite sequence (with finite distinct values)

### Finding the Explicit Formula

- Figure the formula of this sequence

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \dots$$

- Different sequences with same initial values

$$k \geq 0$$

$$a_k = 2k + 1$$

$$b_k = (k - 1)^3 + k + 2$$

## Summation & Product Notation

- Sum of Items Specified

$$\sum_{k=1}^6 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

- Product of Items Specified

$$\prod_{k=1}^5 2k = 2(1) * 2(2) * 2(3) * 2(4) * 2(5)$$

## Variable ending point

- n as the index of the final term

$$\sum_{k=0}^n \frac{k+1}{n+k}$$

- for n = 2
- for n = 3

## Nesting of Sum/Product Notation

- Variations (same or different??):

$$\sum_{j=1}^J \sum_{i=1}^{n_j} Y_{ij}^2 \quad \sum_{j=1}^J (\sum_{i=1}^{n_j} Y_{ij})^2 \quad (\sum_{j=1}^J \sum_{i=1}^{n_j} Y_{ij})^2$$

## Telescoping Series

$$\sum_{k=1}^n \left( \frac{k}{k+1} - \frac{k+1}{k+2} \right)$$

$$\prod_{i=1}^n \left( \frac{i}{i+1} \right)$$