Matrices (Section 2.7)

- We're going to spend a lecture on this topic in preparation for the next few lectures on relations.
- It turns out that matrices are one convenient data structure for representing relations in memory (there are others, too).
- Matrices play an extremely important role in the mathematical subject called linear algebra.
- We'll introduce some algorithms for doing operations on matrices. This will be a case study in counting how many steps it takes an algorithm do do a task, so in a sense it is extending your experience with counting things.

Definitions

- A $m \times n$ matrix over a set S is a rectangular array $\mathbf{A}$ of elements of S, with m rows and n columns.
- More formally, it is a mapping $\mathbf{A} : \{1, \ldots, m\} \times \{1, \ldots, n\} \to S$.
- We write a_{ij} for the row i, column j element of $\mathbf{A}$. We can also write $\mathbf{A} = [a_{ij}]$ to indicate the whole array.
- We write $[a_{i1}, \ldots, a_{in}]$ for the ith row of $\mathbf{A}$, and

$$\begin{bmatrix} a_{1j} \\ \cdots \\ a_{mj} \end{bmatrix}$$

  for the jth column of $\mathbf{A}$.
- Notice that the rows and columns of matrices are themselves matrices.

Examples

- Usually the elements of a matrix are numbers; reals, integers, or natural numbers.
- So, for example,

$$\begin{bmatrix} 3.2 & \sqrt{2} \\ -5 & 7 \\ 1 & 4.0138 \end{bmatrix}$$

  is a 3×2 matrix of reals.

- The elements don't actually have to be numbers, though. You can put other types of things there, as long as they have an "addition" and a "multiplication" defined for that type. For example, you can have elements from the set $\{0, 1\}$, where "addition" is $\vee$, and "multiplication" is $\wedge$, as defined by the truth tables for these operations. You can also have *set-valued* matrices, where the elements of the matrix are subsets of some set X, and where addition is union, and multiplication is intersection. Thus if $X = \{a, b, c\}$,

$$\begin{bmatrix} \{a, b\} & \{b, c\} \\ \{a\} & \{a, b, c\} \\ \emptyset & \{a\} \end{bmatrix}$$

  is a set-valued array.