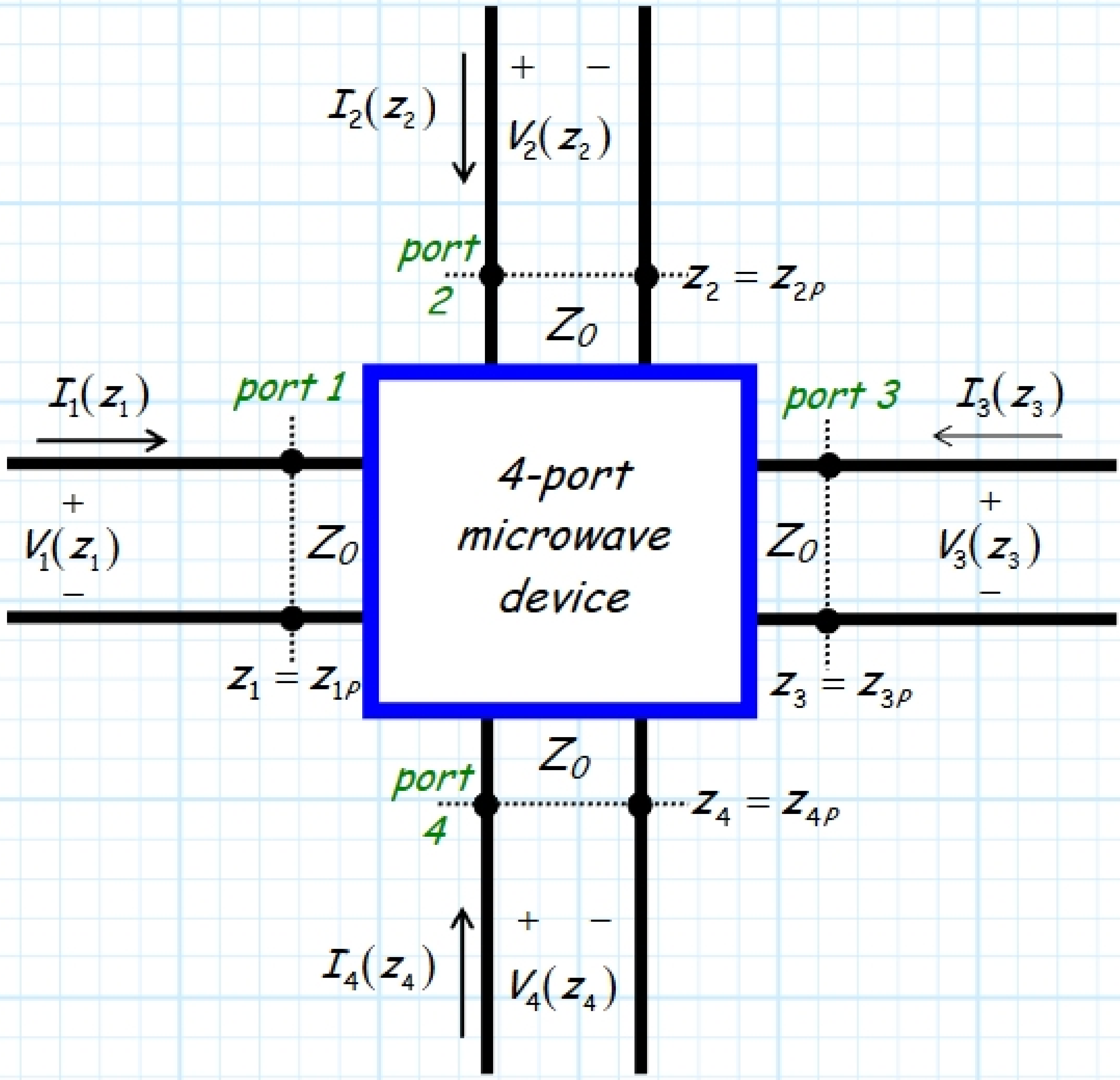
The Impedance Matrix

Consider the **4-port** microwave device shown below:



Note in this example, there are four **identical** transmission lines connected to the same "box". Inside this box there may be a very **simple** linear device/circuit, **or** it might contain a very large and **complex** linear microwave system.

→ Either way, the "box" can be fully characterized by its **impedance matrix**!

First, note that each transmission line has a specific **location** that effectively defines the **input** to the device (i.e., z_{1P}, z_{2P}, z_{3P}, z_{4P}). These often arbitrary positions are known as the **port** locations, or port **planes** of the device.

Thus, the **voltage** and **current** at port n is:

$$V_n(z_n = z_{nP}) \qquad\qquad I_n(z_n = z_{nP})$$

We can **simplify** this cumbersome notation by simply **defining** port n current and voltage as I_n and V_n:

$$V_n = V_n(z_n = z_{nP}) \qquad\qquad I_n = I_n(z_n = z_{nP})$$

For **example**, the current at port **3** would be $I_3 = I_3(z_3 = z_{3P})$.

Now, say there exists a non-zero current at **port 1** (i.e., $I_1 \neq 0$), while the current at all **other** ports are known to be **zero** (i.e., $I_2 = I_3 = I_4 = 0$).

Say we measure/determine the **current** at port **1** (i.e., determine I_1), and we then measure/determine the **voltage** at the port **2** plane (i.e., determine V_2).

The complex ratio between V_2 and I_1 is know as the **trans-impedance parameter** Z_{21}:

$$Z_{21} = \frac{V_2}{I_1}$$

Likewise, the trans-impedance parameters Z_{31} and Z_{41} are:

$$Z_{31} = \frac{V_3}{I_1} \qquad \text{and} \qquad Z_{41} = \frac{V_4}{I_1}$$

We of course could **also** define, say, trans-impedance parameter Z_{34} as the ratio between the complex values I_4 (the current into port 4) and V_3 (the voltage at port 3), given that the current at all other ports (1, 2, and 3) are zero.

Thus, more **generally**, the ratio of the current into port n and the voltage at port m is:

$$Z_{mn} = \frac{V_m}{I_n} \qquad \text{(given that } I_k = 0 \text{ for all } k \neq n\text{)}$$