

General Physics - E&M (PHY 1308) Lecture

Notes

Lecture 004: Continous Charge Distributions and Matter in Electric Fields

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no tags

Goals

- Discuss the transition from finite distributions of charge to continuous distributions of charge
- Discuss what happens to matter in the presence of electric fields

Continuous Charge Distributions

We have been dealing with systems with one or two charges, or treating more complex systems (e.g. the heart, on the problem set) like a single point charge. Today, we get tough. Today, we discuss the transitions from few charges to lots of charges.

Nature is build from elementary charges like electrons and protons. However, a typical number in a volume of air or in a solid is Avagadro's Number: 6.02×10^{23} ! Good luck adding all of those forces and fields together. We need tools to attack these more realistic situations. We need calculus.

To begin, let's discuss the bulk properties of continuous charge distributions:

- *Volumes of charge*: given a volume (3 dimensions) of charge, we will speak of the **volume charge density**, ρ , whose units are charge per unit volume, C/m^3 .
- *Surfaces of charge*: if the charge is spread out over a surface, instead of throughout a volume, we will speak of the **surface charge density**, σ , whose units are C/m^2 .

- *Line of charge*: if the charge can be described as distributed along a single dimension, we will then speak of **linear charge density**, λ , whose units are C/m .

Using these concepts lets us summarize the charge distribution properties of an extended, continuous system. Instead of saying a charge is here, at some coordinates, and over there, at some other coordinates, we speak instead of densities of charge along a line, surface, or throughout a volume.

Cement the concept: examples

Imagine that I have a cube, whose sides are each of length 1.0m. In each of the following situations, tell me the appropriate bulk property of the system:

- What is the bulk property of the system if there is a charged object inside the cube whose strength is 10C? ANSWER: we should be concerned about the charge per unit volume, or the volume charge density of the system. In that case, it will be $\rho = (10C)/(1m)^3 = 10C/m^3$.
- What is the bulk property of the system if the same 10C charge is distributed over the sides of the cube? ANSWER: we should be concerned about the charge per unit area, or the surface charge density. In that case, we need the surface area of the cube. That's $A = 6 \times (1m^2) = 6m^2$. Thus $\sigma = 10C/6m^2 = (5/3)C/m^2$.
- What is the bulk property of the system if the same 10C charge is distributed only along the thin corners at each side of the cube? ANSWER: if the charge is isolated to just the corners where each side meets, you can imagine the cube as being made from a wireframe where each wire holds part of the total charge. Thus, the bulk property we are concerned with is the linear charge density. We need to know the lengths of these "wires". There are 12 such lines that make up the cube. Thus the total length is $L = 12 \times 1m = 12m$, and $\lambda = 10C/12m = (5/6)C/m$.

Summing the fields

We've used the Principle of Superposition to add together individual fields from a series of charges. Now that we're dealing with lots of charge spread

out over lines, surfaces, and volumes, we need a better way to do the sums. Calculus - and specifically the integral - gives us this power.

Recall that an integral is just given in the limit of a sum of very small pieces of a problem. For instance, if we have a bunch of small charges and we want to find the total field at some point, P. Each infinitesimal unit of charge, dq , is responsible for emitting its own infinitesimal piece of the total field, $d\vec{E}$. Thus to obtain the total field we must integrate over the individual infinitesimal fields:

$$\vec{E} = \int d\vec{E}$$

We know how to write the electric field of each infinitesimal point of charge - that's just using Coulomb's Law! We have to do a transformation of variable in the integral, from $d\vec{E}$ to dq . That's simply done as follows:

$$d\vec{E} = \left(\frac{d\vec{E}}{dq} \right) dq = \left(\frac{d}{dq} \frac{kq}{r^2} \hat{r} \right) dq = \left(\frac{k}{r^2} \hat{r} \right) \frac{dq}{dq} dq = \frac{k dq}{r^2} \hat{r}$$

Thus:

$$\vec{E} = \int \frac{k dq}{r^2} \hat{r}$$

This is the easy part. The hard part is being given a physical problem and being asked to solve for the field in terms of the geometry of that problem. Attacking real problems will involve translating the geometry of the problem into something over which you can then integrate more easily. Let's look at an example.

Example: a line of charge

Consider a thin wire that carries a charge. The bulk property of the wire that we are interested in is the linear charge density, λ . Imagine that this wire - perhaps a power line or an electric cable of some other sort - lies along the x-axis. How do we find the electric field at some point P away