Modern Physics (PHY 3305) Lecture Notes

Velocity, Energy and Matter (Ch. 2.6-2.7)

SteveSekula, 29 January 2010 (created 13 December 2009)

tags: lecture

CHAPTERS 2.6-2.7

Review of last lecture

- We explored one of the popular so-called "paradoxes" of special relativity and realized that there is no paradox when Einstein's postulates are applied thoroughly to the question
- We explored kinematics in special relativity - motion in a frame of reference - beginning with the Doppler effect for light.

Today

- Briefly discuss the transformation of velocities
- Challenge classical notions of momentum and energy and discuss the implications of new realities for society
- We will begin a discussion of the wave nature of light.

Special Relativistic Transformation of Velocities

One of the most stunning revelations of special relativity proceeds from the vanilla-sounding "transformation of velocities." Recall that the Galilean transformation told us:

$$\text{FRAME } S: \; u = u' + v$$

$$\text{FRAME } S': \; u' = u - v$$

Special relativity seems to be a better description of the relationship between space and time. What is the "correct" transformation from the perspective of special relativity?

Remember:

$$u = dx/dt$$

and

$$dx = \frac{\partial x}{\partial x'}dx' + \frac{\partial x}{\partial t'}dt'$$

From those relationships, we can derive how the motion of objects in one frame are related to the motions in another frame. For instance, in frame S:

$$u' = \frac{u - \nu}{1 - \frac{\nu u}{c^2}}$$

In Frame S':

$$u = \frac{u' + \nu}{1 + \frac{\nu u'}{c^2}}$$

The Mathematical Bag of Tricks: Binomial Expansion

We keep seeing things that look like $(a + x)^n$, like:

$$\gamma_\nu = \frac{1}{\sqrt{1 - (\nu/c)^2}} \equiv (1 - \beta^2)^{-1/2},$$

or

$$1/\gamma_\nu = (1 - \beta^2)^{+1/2},$$

or in the Doppler Shift

$$(1 + \nu/c)^{-1}.$$

All of these are variations on

$$(a + x)^n.$$

When you see that, and your calculator fails you because $x << 1$, you apply the Binomial Expansion:

$$f(x) = (a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + ...$$

where $(x^2 < a^2)$

Here are some useful cases:

$$\frac{1}{(1-x^2)} = 1 + x^2 + x^4 + x^6 + ...$$

$$\sqrt{\frac{1}{(1-x^2)}} = 1 + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{16}x^6 + ...$$

This will be helpful for you when, for instance, you are faced with calculating relativistic effects for velocities much smaller than that of light, but where the relativistic effects, though tiny, can have a large impact on the outcome.

Momentum and Energy

Remember the classical conservation of momentum:

$$\sum \vec{p}_{initial} = \sum \vec{p}_{final}$$

What about when you apply the classical transformation of velocities - that is, is momentum conserved in classical physics using the Galilean Transformation?

The answer is yes. If anyone would like to see this, consider a situation where two objects of mass m_1 and m_2 collide head-on. In frame S, their initial momentum and final momentum is given by

$$m_1 u_1^i + m_2 u_2^i = m_1 u_1^f + m_2 u_2^f.$$

In another frame, S', moving along the direction of the collision at speed ν, the form of the conservation equation for the collision should be:

$$m_1 u_1'^i + m_2 u_2'^i = m_1 u_1'^f + m_2 u_2'^f$$