Network Observational Methods and Quantitative Metrics: II

- Topics
  - Degree correlation
  - Exploring whether the sign of degree correlation can be predicted from network type or similarity to regular structures, or details about the network itself, or maybe nothing
  - Calculating degree correlation for simple regular structures like trees and grids

Summary Properties of Several Big Networks (Newman)

	Network	Type	n	m	z	l	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
Social	Film actors	Undirected	449, 913	25, 516, 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	Company directors	Undirected	7, 673	55, 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	Math coauthorship	Undirected	253, 339	496, 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	Physics coauthorship	Undirected	52, 909	245, 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	Biology coauthorship	Undirected	1, 520, 251	11, 803, 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	Telephone call graph	Undirected	47, 000, 000	80, 000, 000	3.16		2.1				8, 9
	Email messages	Directed	59, 912	86, 300	1.44	4.95	1.5/2.0		0.16		136
	Email address books	Directed	16, 881	57, 029	3.38	5.22	–	0.17	0.13	0.092	321
	Student relationships	Undirected	573	477	1.66	16.01	–	0.005	0.001	-0.029	45
	Sexual contacts	Undirected	2, 810				3.2				265, 266
Information	WWW.nd.edu	Directed	269, 504	1, 497, 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	14, 34
	WWW.Altavista	Directed	203, 549, 046	2 ,130, 000, 000	10.46	16.18	2.1/2.7				74
	Citation network	Directed	783, 339	6, 716, 198	8.57		3.0/–				351
	Roget's thesaurus	Directed	1, 022	5, 103	4.99	4.87	–	0.13	0.15	0.157	244
	Word co-occurrence	Undirected	460, 902	17, 000, 000	70.13		2.7		0.44		119, 157
Technological	Internet	Undirected	10, 697	31, 992	5.98	3.31	2.5	0.035	0.39	-0.189	86, 148
	Power grid	Undirected	4, 941	6, 594	2.67	18.99	–	0.10	0.080	-0.003	416
	Train routes	Undirected	587	19, 603	66.79	2.16	–		0.69	-0.033	366
	Software packages	Directed	1, 439	1, 723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
	Software classes	Directed	1, 377	2, 213	1.61	1.51	–	0.033	0.012	-0.119	395
	Electronic circuits	Undirected	24, 097	53, 248	4.34	11.05	3.0	0.010	0.030	-0.154	155
	Peer-to-peer network	Undirected	880	1, 296	1.47	4.28	2.1	0.012	0.011	-0.366	6, 354
Biological	Metabolic network	Undirected	765	3, 686	9.64	2.56	2.2	0.090	0.67	-0.240	214
	Protein interactions	Undirected	2, 115	2, 240	2.12	6.80	2.4	0.072	0.071	-0.156	212
	Marine food web	Directed	135	598	4.43	2.05	–	0.16	0.23	-0.263	204
	Freshwater food web	Directed	92	997	10.84	1.90	–	0.20	0.087	-0.326	272
	neural network	Directed	307	2, 359	7.68	3.97	–	0.18	0.28	-0.226	416, 421

Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex-vertex distance l; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, See. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Image by MIT OpenCourseWare.

Table 2 in Mark E. J. Newman's. "The Structure and Function of Complex Networks." *SIAM Review* 45, no. 2 (2003): 167-256.

Community-finding and Pearson Coefficient r

- Newman says technological networks seem to have $r < 0$ while social networks seem to have $r > 0$
- Newman and Park sought an explanation in community structure and clustering: "Why Social Networks are Different From Other Kinds of Networks" *Phys Rev E*, **68**, 036122 (2003)
- Social networks can arise by people joining multiple groups and generating multiple connections
- Networks derived from these multiple connections have positive *r*
- Networks coauthconn and coauthwhole are from this paper
  - coauthconn is the connected portion with 147 nodes
  - coauthwhole has 42 clusters, smallest has 2 nodes, biggest has 5