

Filters

A RF/microwave **filter** is (typically) a passive, reciprocal, 2-port linear device.



If port 2 of this device is terminated in a **matched** load, then we can relate the incident and output power as:

$$P_{out} = |S_{21}|^2 P_{inc}$$

We define this power transmission through a filter in terms of the **power transmission coefficient T**:

$$T \doteq \frac{P_{out}}{P_{inc}} = |S_{21}|^2$$

Since microwave filters are typically **passive**, we find that:

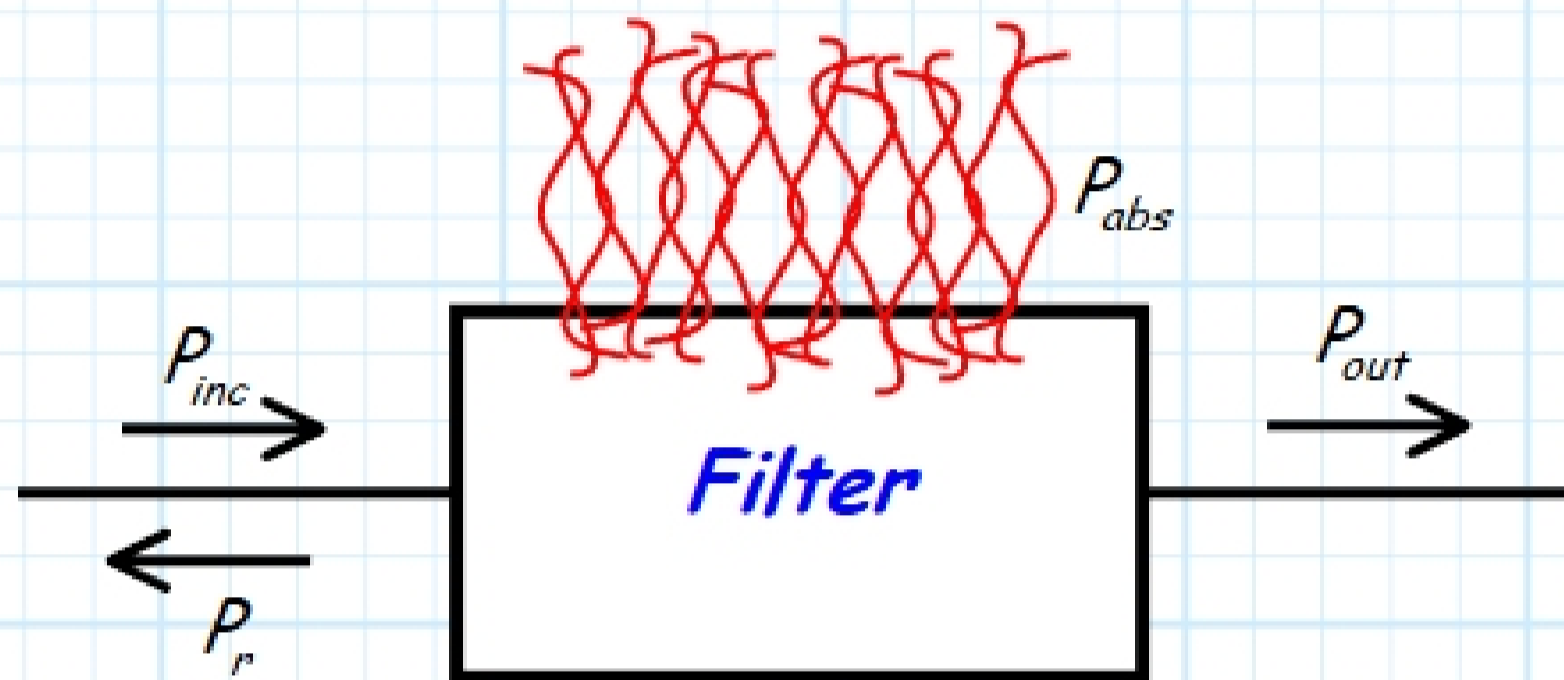
$$0 \leq T \leq 1$$

in other words, $P_{out} \leq P_{inc}$.

Q: What happens to the "missing" power $P_{inc} - P_{out}$?

A: Two possibilities: the power is either **absorbed** (P_{abs}) by the filter (converted to heat), or is **reflected** (P_r) at the input port.

I.E.:



Thus, by conservation of energy:

$$P_{inc} = P_r + P_{abs} + P_{out}$$

Now **ideally**, a microwave filter is lossless, therefore $P_{abs} = 0$ and:

$$P_{inc} = P_r + P_{out}$$

which **alternatively** can be written as:

$$\frac{P_{inc}}{P_{inc}} = \frac{P_r + P_{out}}{P_{inc}}$$

$$1 = \frac{P_r}{P_{inc}} + \frac{P_{out}}{P_{inc}}$$

Recall that $P_{out}/P_{inc} = \mathbf{T}$, and we can likewise define P_r/P_{inc} as the **power reflection coefficient**:

$$\Gamma \doteq \frac{P_r}{P_{inc}} = |S_{11}|^2$$

We again emphasize that the filter output port is terminated in a **matched load**.

Thus, we can conclude that for a **lossless filter**:

$$1 = \Gamma + \mathbf{T}$$

Which is simply **another** way of saying for a lossless device that $1 = |S_{11}|^2 + |S_{21}|^2$.

Now, **here's** the important part!

For a microwave **filter**, the coefficients Γ and \mathbf{T} are **functions of frequency!** I.E.,:

$$\Gamma(\omega) \quad \text{and} \quad \mathbf{T}(\omega)$$

The **behavior** of a microwave filter is described by these **functions!**