PH314 Midterm #1

4 October 2007

Closed book

$c = 3\times10^{10}$ cm s^{-1}	speed of light
$G = 6.7\times10^{-8}$ gm^{-1} cm^3 s^{-2}	universal constant of gravitation
$k = 1.38\times10^{-16}$ erg K^{-1}	Boltzmann's constant
$M_\odot = 2\times10^{33}$ gm	solar mass
$L_\odot = 3.9\times10^{33}$ ergs s^{-1}	solar luminosity
$r_{AU} = 1.5\times10^{13}$cm	Astronomical Unit (distance to the sun)
$r_{Moon} = 3.8\times10^{10}$cm	Earth-Moon distance
$M_\oplus = 6\times10^{27}$ gm	mass of the Earth
$m_p = 1.67\times10^{-24}$ gm	mass of the proton

1. The bolometric magnitude of the sun is $M_\odot = 4.53$.
(a) A Type Ia supernova has a peak luminosity of 10^{43} ergs s^{-1}. What is the absolute magnitude of this supernova?
(b) There are roughly 2.5 billion stars in the Large Magellanic Cloud, our nearest neighbor galaxy. If all the stars were like the sun, what would the absolute magnitude of the LMC be? Compare this to (a)

2. The center of our Galaxy is 8.5kpc distant.
(a) What is the parallax of a star at the Galaxy Center? Remember that $\theta=1/p$ where p is the distance in parsecs and θ is the parallactic angle in arcseconds.
(b) Compare this with the resolved image size of a star imaged by the Hubble Space Telescope (HST), assuming the diameter of the telescope is D=2.5m. Hint: the resolved image size of a point of light in a telescope is given by $\theta = 1.22 \times 205263 \times \lambda/D$ in units of arcseconds. Assume λ=500nm=5×10^{-7}m which is the wavelength of visible light.
(c) It is easy for a telescope to measure an image motion that is 2% of the image size. Can the HST measure the parallax of a star in the Galactic Center?

3. (a) The minimum mass for a gas cloud to collapse into a star is called the Jeans mass. From the virial theorem, $2K + U = 0$ for an object in equilibrium. If $2K < U$, then the thermal energy cannot hold up the cloud and it will collapse. Show that the Jeans mass goes as $M_J = (5kT_0/[Gm_p])^{3/2} \times (3/[4\pi\rho_0])^{1/2}$ where ρ_0 is the initial density and T_0 is the initial temperature of the cloud.

Hint: the gravitational potential is $U = -3GM_c^2/5R_c$, where M_c and R_c are the mass and radius of the collapsing cloud. The total kinetic energy is just $K=3/2NkT$ where N is the total number of particles in the cloud. Assume that the cloud is just hydrogen so that $N=M_c/m_p$. Equate 2K with –U and note that the mean density of the cloud is $\rho_0 = M_c/[4\pi R_c^3/3]$.

(b) If I plug in the numbers for you, I find M_J (gm) = 1.84×10^{35}gm x $T_0^{3/2} n_0^{-1/2}$ where n_0 is the number density ($\rho_0 = n_0 m_p$). Typical values for interstellar clouds are T_0 = 50K, and n_0 = 500 per cm3. What is the Jeans mass for star formation under these conditions? Compare this to the largest mass for a pure hydrogen star of $500M_\odot$.

4. Suppose you are in space and you see a black hole coming at you. It is only 9mm across. Your friend says "Don't worry, it is real small." You on the other hand are not sure. Calculate the mass of the black hole and compare it to the Earth's mass. Hint: The total energy of a gravitating systems is E_T = kinetic energy + potential energy = $1/2\ mv^2 + (-GmM/R)$. Assume you have a black hole when the escape velocity from the surface of the black hole to infinity is c, the speed of light, and the total energy is 0 at infinity.

5. (a) The diameter of the sun is 32 arcminutes. Deduce the linear size of the sun from this angle and the value of the AU above.
(b) Calculate the effective temperature from the solar radius and the luminosity of the sun (given above).
Remember: $\tan\theta = \sin\theta = \theta$ for small angles when the angle is in radians. Convert from radians to arcmintues using the fact that in 2π radians, there are 360 degrees x 60 arcminutes. Also, remember that the luminosity = surface area times the flux per unit area. The latter is given by σT^4.

6. The angular momentum of the Moon around the Earth is roughly M_{Moon} x orbital velocity x distance from the Earth.
(a) Using Kepler's relation (the so-called "Kepler's Third Law") between the distance and period of revolution, show that the angular momentum of the Moon goes as M_{Moon} x $r_{Moon}^{1/2}$ where r_{Moon} is the Earth-Moon distance and M_{Moon} is the mass of the Moon. The rotation period of the sun is roughly a month.
(b) The sidereal period of the Moon is about 28 days. The tidal forces on the Earth Moon system have forced the Moon to spin on its axis at the same period as its orbit period (28 days). Similarly these tidal forces are also forcing the Earth and Moon to converge to having the same periods for the Earth's spin and the Moon's orbital period. As the Earth slows its rotation period down from 24 hours, what will happen to the Moon's distance from the Earth? Hint: consider that total angular momentum of the Earth Moon system.

7. (a) Explain in words the principle of hydrostatic equilibrium in a star between the gravitational pressure and the thermal pressure of a star.
(b) In a white dwarf, the degeneracy of non-relativistic electrons produces a pressure of $P_{e-} \sim (\rho)^{5/3}$, where ρ is the mass density. The gravitational pressure is $P_g \sim GM^2/R^4$. Show that $R \sim M^{-1/3}$.

8. The distance modulus is given as $m-M = 5\log_{10}d - 5$ where d is the distance in parsecs. Assuming the absolute magnitude of the sun is M=4.73, what is the distance (in kpc) we can see a solar like star at the detection limit of the HST of m=30? Give the answer in Mpc.

9. In a paragraph or two, tell me how you feel about spending money on returning to the moon.