

MATH 251

MIDTERM EXAMINATION I
PENN STATE UNIVERSITY

February 20, 2002

There are 5 multiple choice and 7 partial credit problems in this examination. Each multiple choice problem has 5 choices. Circle the correct answer. Each multiple choice problem is worth 5 points.

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK YOUR EXAMINATION BOOKLET CAREFULLY.
THERE SHOULD BE 12 PROBLEMS ON 10 PAGES.

1. (5 points) Which of the following is TRUE?

- a) $y' = \frac{t}{y}$ is a first order linear differential equation
- b) $\sin t y'' + (1 - t^2)y' + \cos t y = 0$ is a second order linear differential equation
- c) $y'' + (y')^3 + y = 0$ is a nonlinear differential equation of order 3
- d) $y'' + y' + y = t$ is a second order homogeneous differential equation
- e) $\frac{\partial y}{\partial t} + ty = 0$ is an ordinary differential equation

2. (5 points) Let $y(t)$ be the solution of the initial value problem

$$\begin{aligned}\frac{dy}{dx} &= y^3 - y \\ y(0) &= \frac{9}{10}.\end{aligned}$$

Then $\lim_{t \rightarrow \infty} y(t)$ is equal to

- a) ∞
- b) $\frac{9}{10}$
- c) 1
- d) 0
- e) -1

3. (5 points) Find the value for the constant b , for which given equation is exact.

$$(e^x \sin y + bx^2y^2)dx + (e^x \cos y + x^3y)dy = 0$$

- a) $b = 0$
 - b) $b = \frac{1}{3}$
 - c) $b = 3$
 - d) $b = \frac{3}{2}$
 - e) $b = 1$
4. (5 points) Let $y_1(t)$ and $y_2(t)$ be two solutions of a second order, homogeneous, linear differential equation. Suppose the Wronskian $W(y_1(t), y_2(t)) = e^{-t}$. Which of the following is FALSE?
- a) $y_1(t)$ and $y_2(t)$ are linearly independent functions.
 - b) $2y_1(t) - 3y_2(t)$ is also a solution of the differential equation.
 - c) $y_1(t)$ and $y_2(t)$ do not constitute a fundamental set of solutions.
 - d) All solutions of the differential equation can be expressed as $c_1y_1(t) + c_2y_2(t)$, where c_1 and c_2 are constants.
 - e) $W(2y_1(t), 3y_2(t)) = 6e^{-t}$