

**Midterm Exam No. 2 - *Answers***  
**July 29, 2002**

Answer all questions, in blue book. Plan ahead and budget your time. The questions are worth a total of 60 points, as indicated. You will have 80 minutes to complete the exam.

1. [19 points] In the diagram on page 2 are drawn unit-value isoquants for a world that produces three goods,  $X_1$ ,  $X_2$ , and  $X_3$ , using two factors,  $K$  and  $L$ , at free trade prices  $p_1$ ,  $p_2$ , and  $p_3$  measured in dollars. To assist you, I have drawn two straight lines tangent to pairs of the isoquants, showing the points of tangency as open dots, and also drawing the rays from the origin through these points of tangency. Factor endowments are shown for seven countries, represented by the solid dots labeled  $E^1, \dots, E^7$ . Answer the following questions about these countries. Feel free to tear off this front page of the exam so that you can look at it and the diagram together.

- a. (4 points) Which country (or countries, if there are ties) has the largest absolute stock of capital? Which has the most labor? Which countries have the largest and the smallest *ratios* of capital to labor in their endowments?

*Country 2 has the largest capital stock,  $K_2$ .*

*Country 7 has the largest labor force,  $L_7$ .*

*Country 1 has the largest capital-labor ratio,  $k_1 = K_1/L_1$  (since it is to the left of the steepest ray drawn, and all other countries are to the right of that).*

*Country 7 has the smallest capital-labor ratio.*

- b. (4 points) Which country (or countries) has the highest rental price of capital? Which has the lowest? Which pairs or groups of countries, if any, have the same wage rate of labor?

*Countries 6 and 7 both have the highest rental price of capital,  $r_6=r_7$ .*

*Country 1 has the lowest rental price of capital,  $r_1$ .*

*Factor price equalization holds in the two cones, and therefore countries 2 and 4 have the same wage of labor,  $w_2=w_4$ , and so do countries 6 and 7,  $w_6=w_7 (<w_2)$ .*

- c. (2 points) Which country produces the most good 1? Which produces the most good 3?

*Only country 7 produces good 1 at all, so it must produce the most. (It employs the vector of factors  $v_1^1$  in producing good 1.)*

*Countries 1, 2, and 4 all produce good 3, country 1 producing only that, while countries 2 and 4 produce both good 3 and good 2. They employ vectors  $v_3^2$  and  $v_3^4$  in producing good 3. Clearly, country 2 employs more of both factors in the sector than either of the other countries, so it produces the most good 3.*

- d. (4 points) What does country 6 produce, what does it export and import, and how do its factor prices compare to factor prices in the other countries?

*Country 6 on the edge of a cone. It produces only good 2 (since to produce any of good 1 would leave it the wrong amount of factors to fully employ in good 2). It therefore exports good 2, while it imports goods 1 and 3. Its factor prices are the same as country 7's,  $r_7$  and  $w_7$ , and thus its wage is lower than in all of countries 1-5 and its rental on capital is higher than in countries 1-5.*

- e. (2 points) In what industry does country 7 employ the largest fraction of its total labor force? Is country 7's output of good 1 worth more than \$1, less than \$1, or exactly \$1?

*Country 7 employs vector  $v_2^7$  in sector 2 and  $v_1^7$  in sector 1, from which it employs more labor in sector 2 than in sector 1, hence a larger fraction of its labor force. Country 7's inputs to sector 1 lie well inside its unit isocost line (the straight line from  $1/r_7$  to  $1/w_7$ , and therefore these inputs, and also its output, are worth less than \$1*

- f. (3 points) Which country or countries (if any) produce all three goods? In what direction would the price of good 2 have to change (holding the prices of goods 1 and 3 constant) in order to make it possible for some country that currently does not produce all three goods to do so? If that price change happened, would factor prices then be equal in all seven countries?

*No country produces all three goods. In a 2-cone equilibrium such as this, prices are such that no country can produce all three goods in free trade. To change that, all three unit-value isoquants would have to be tangent to a single straight line. To achieve that by changing only  $p_2$ , we would need to reduce  $p_2$  until its unit-value isoquant had shifted out to the new position shown in the lower diagram below. There would then be the single cone shown, instead of two cones, and countries 1 and 2 would lie outside of it. So factor prices would not be equal across all seven countries. (Indeed, factor prices in country 1 would continue to be what they were before.)*

