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Calculus III 2210-4
Sample Midterm Exam 1
Exam Date: Wed 28 Sept 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Vector calculus)

(a) Let $\mathbf{r}(t) = \begin{pmatrix} e^{2t} \\ e^{-t} \end{pmatrix}$. Define $\mathbf{u}(t) = r\mathbf{r}(t) + (\mathbf{r}(t) \cdot \mathbf{r}'(t))\mathbf{r}(2t + 10)$. Find $\mathbf{u}'(0)$.

(b) Let $\mathbf{r}(t) = \begin{pmatrix} e^{2t} \\ e^{-t} \\ e^{3t} \end{pmatrix}$. Find $\int_0^{\ln 2} \mathbf{r}(t) dt$.

(c) State and prove a formula for the derivative of the cross product of two vector functions $\mathbf{u}(t)$ and $\mathbf{v}(t)$.

(d) True or false: $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$. Justify.

(e) True or false: $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{v})\mathbf{w} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$. Justify.

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2. (Vector algebra)

- (a) Prove that orthogonal vectors \mathbf{u} , \mathbf{v} satisfy $|\mathbf{u}|^2 + |\mathbf{v}|^2 = |\mathbf{u} + \mathbf{v}|^2$.
- (b) Report three different vectors orthogonal to $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.
- (c) Let \mathbf{u} and \mathbf{v} have dot product zero. Prove that they are 90 degrees apart.
- (d) For which vectors \mathbf{u} and \mathbf{v} is it true that $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$? Justify.
- (e) State the five dot product properties.
- (f) State the eight properties in the vector toolkit.

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3. (Differential geometry)

- (a) Prove that a curve $y = f(x)$ with $f''(x) = 0$ has zero curvature.
- (b) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the velocity and acceleration at $t = \ln 2$.
- (c) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the unit tangent, principal normal, binormal and curvature at $t = \ln 2$.
- (d) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the tangential and normal components of acceleration at $t = \ln 2$.

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