Math 2860 Midterm

1.) A tank is filled with 80 gallons of a water/salt mixture with an initial amount of salt (Q_0 lbs) mixed into it. 4 gallons/min of a solution with a ¼ lbs per gallon salt concentration is pumped into the tank, while 4 gallons/min of the entire tank's mixture is drained out.

a.) Write a differential equation for Q'(t) where Q(t) is the amount of salt in the solution after t minutes. (8 pts.)

b.) What is the equilibrium solution for the equation in part a? (8 pts.)

c.) If $Q_0 = 40$ lbs, solve the initial value differential equation. (8 pts.)

2.) Give the general solution to: $y' + \frac{2}{x} y = \frac{\sin x}{x^2}$ (10 pts.)

3.) Find the general solution to: $y'' - 12y' + 32y = 0$ (8 pts.)

4.) Find the general solution to: $y'' - 2y' + 10y = 0$ (8 pts.)

5.) Find the general solution to: $y'' - 8y' + 16y = 0$ (8 pts.)

6.) Consider: $x^2y'' + 4xy' + 2y = 0$, and observe that $y_1 = \frac{1}{x}$ is a solution.

a.) Use Abel's theorem to find the Wronskian. (Make the y" coefficient = 1) (8 pts.)

b.) Using the definition of the Wronskian along with your solution in part a, find y_2. (10 pts.)