

Midterm 2

Closed book exam. One 8.5x11" study sheet is allowed. A calculator is allowed also.
Exam is worth 30 points, 15% of your total grade.

Forms of the Schrodinger equation:

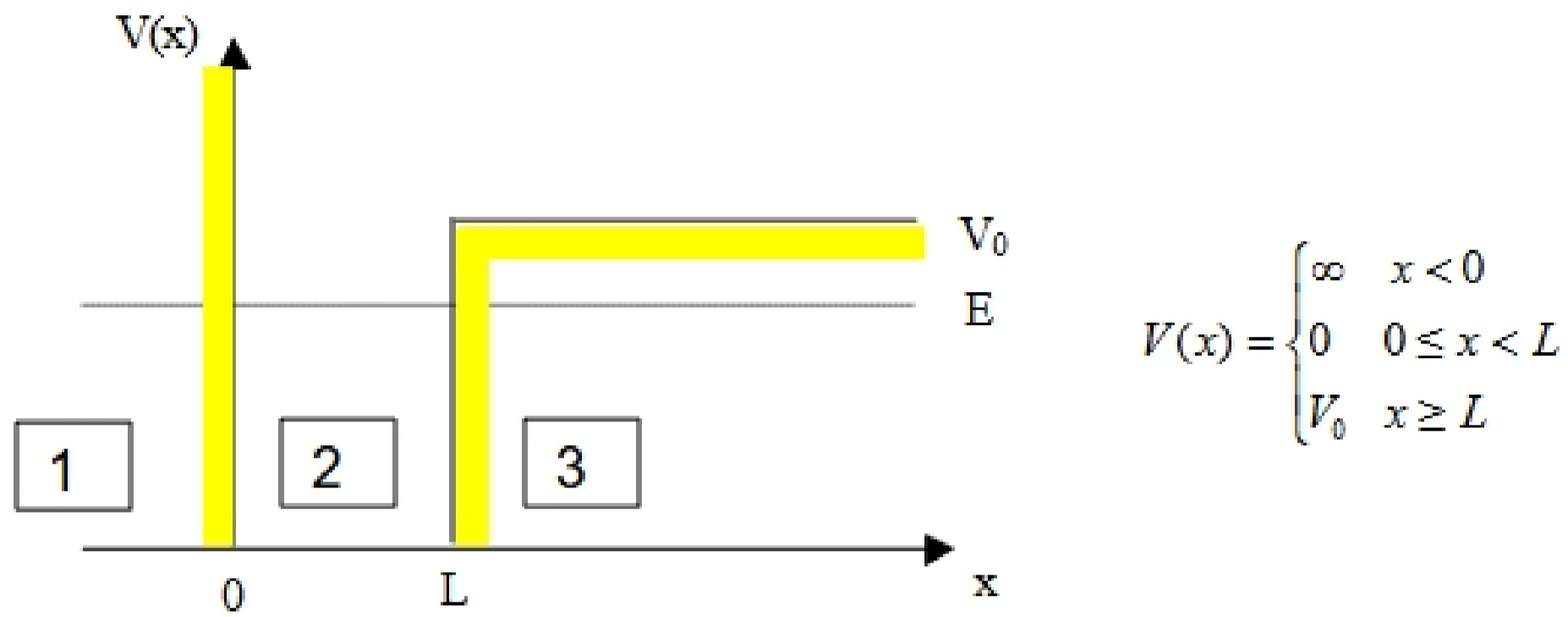
$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x,t)\psi = i\hbar \frac{\partial \psi}{\partial t} \qquad -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x)\psi = E\psi$$

Constants and equations:

$h = 6.6261 \cdot 10^{-34} \text{ J-s}$	$c = 3.0 \cdot 10^8 \text{ m/s}$	$e = 1.6022 \cdot 10^{-19} \text{ C}$
$h = 4.1357 \cdot 10^{-15} \text{ eV-s}$	$\lambda v = c$	$1 \text{ eV} = 1.6022 \cdot 10^{-19} \text{ J}$
$hc = 1240 \text{ eV-nm}$	$E_\gamma = h\nu$	$1 \text{ MeV} = 10^6 \text{ eV}$
$\hbar = h/2\pi = 1.0546 \cdot 10^{-34} \text{ J-s}$	$\lambda = h/p$	$1 \text{ nm} = 10^{-9} \text{ m}$
$\hbar = 6.5821 \cdot 10^{-16} \text{ eV-s}$	$1 \text{ Watt} = 1 \text{ J/s}$	$1 \text{ MHz} = 10^6 \text{ s}^{-1}$
$m_e = 9.11 \cdot 10^{-31} \text{ kg}$	$m_p = 1.673 \cdot 10^{-27} \text{ kg}$	$m_\alpha = 1.881 \cdot 10^{-28} \text{ kg}$
$m_e = 0.511 \text{ MeV} / c^2$	$m_p = 938.3 \text{ MeV} / c^2$	$m_\alpha = 105.7 \text{ MeV} / c^2$
$\frac{e^2}{4\pi\epsilon_0} = 1.44 \cdot 10^{-9} \text{ eV-m}$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \cdot \mathbf{B})$	$W = qV$
$R_\infty = \frac{E_0}{hc} = 1.09737 \cdot 10^7 \text{ m}^{-1}$	$R_H = \frac{E_0}{hc} = 1.09678 \cdot 10^7 \text{ m}^{-1}$	
$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_u^2} \right)$	$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.53 \cdot 10^{-10} \text{ m}$	
$E_n = \frac{-Z^2 e^4 m_e}{2\hbar^2 (4\pi\epsilon_0)^2 n^2} = (-13.6 \text{ eV}) \frac{Z^2}{n^2}$	$\alpha_B = \frac{e\hbar}{2m_e} = 5.7884 \cdot 10^{-5} \text{ eV/T}$	
$\Delta x \Delta p \geq \hbar / 2$	$\Delta E \Delta t \geq \hbar / 2$	

$$\langle f(x) \rangle = \int \psi^*(x) f(x) \psi(x) dx$$

1. Consider the semi-infinite potential well shown below. The kinetic energy E of a particle of mass m is smaller than the barrier height V_0 , so it is trapped in the well.



(a) [2 points] What is the wavefunction solution to the time-independent Schrodinger Equation for region 1 ($x < 0$)?

(b) [4 points] What is the general form of the wavefunction solution for region 2 ($0 \leq x < L$) taking into account the boundary condition at $x = 0$? Express your answer as a function of (kx) , and determine the constant k .

- (c) [4 points] What is the general form of the wavefunction solution for region 3 ($x \geq L$) taking into account the boundary condition at $x = \infty$? Express your answer as a function of (αx) , and determine the constant α .

- (d) [4 points] Determine two relations between the solution of region 2 and the solution of region 3 by applying boundary conditions at $x = L$.