

Midterm 2

Closed book exam. One 8.5x11" study sheet is allowed. A calculator is allowed also.
Exam is worth 30 points, 15% of your total grade.

Forms of the Schrodinger equation:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x, t)\psi = i\hbar \frac{\partial \psi}{\partial t} \qquad -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x)\psi = E\psi$$

Constants and equations:

$h = 6.6261 \cdot 10^{-34} \text{ J-s}$	$c = 3.0 \cdot 10^8 \text{ m/s}$	$e = 1.6022 \cdot 10^{-19} \text{ C}$
$h = 4.1357 \cdot 10^{-15} \text{ eV-s}$	$\lambda f = c$	$1 \text{ eV} = 1.6022 \cdot 10^{-19} \text{ J}$
$hc = 1240 \text{ eV-nm}$	$f = E/h$	$1 \text{ MeV} = 10^6 \text{ eV}$
$\hbar = h/2\pi = 1.0546 \cdot 10^{-34} \text{ J-s}$	$\lambda = h/p$	$1 \text{ nm} = 10^{-9} \text{ m}$
$\hbar = 6.5821 \cdot 10^{-16} \text{ eV-s}$	$1 \text{ Watt} = 1 \text{ J/s}$	$1 \text{ MHz} = 10^6 \text{ s}^{-1}$
$m_e = 9.11 \cdot 10^{-31} \text{ kg}$	$m_p = 1.673 \cdot 10^{-27} \text{ kg}$	$m_\alpha = 1.881 \cdot 10^{-28} \text{ kg}$
$m_e = 0.511 \text{ MeV} / c^2$	$m_p = 938.3 \text{ MeV} / c^2$	$m_\alpha = 105.7 \text{ MeV} / c^2$
$\frac{e^2}{4\pi\epsilon_0} = 1.44 \cdot 10^{-9} \text{ eV-m}$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \cdot \mathbf{B})$	$W = qV$
$R_\infty = \frac{E_0}{hc} = 1.09737 \cdot 10^7 \text{ m}^{-1}$	$R_H = \frac{E_0}{hc} = 1.09678 \cdot 10^7 \text{ m}^{-1}$	
$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_u^2} \right)$	$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.53 \cdot 10^{-10} \text{ m}$	
$E_n = \frac{-Z^2 e^4 m_e}{2\hbar^2 (4\pi\epsilon_0)^2 n^2} = (-13.6 \text{ eV}) \frac{Z^2}{n^2}$	$\alpha_B = \frac{e\hbar}{2m_e} = 5.7884 \cdot 10^{-5} \text{ eV/T}$	

$$\Delta x \Delta p \approx \hbar$$

$$\Delta E \Delta t \approx \hbar$$

$$\langle \hat{f} \rangle = \int \psi^*(x) \hat{f} \psi(x) dx \qquad \int_0^\infty x^n e^{-x/\alpha} dx = n! \alpha^{n+1} \qquad n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$$

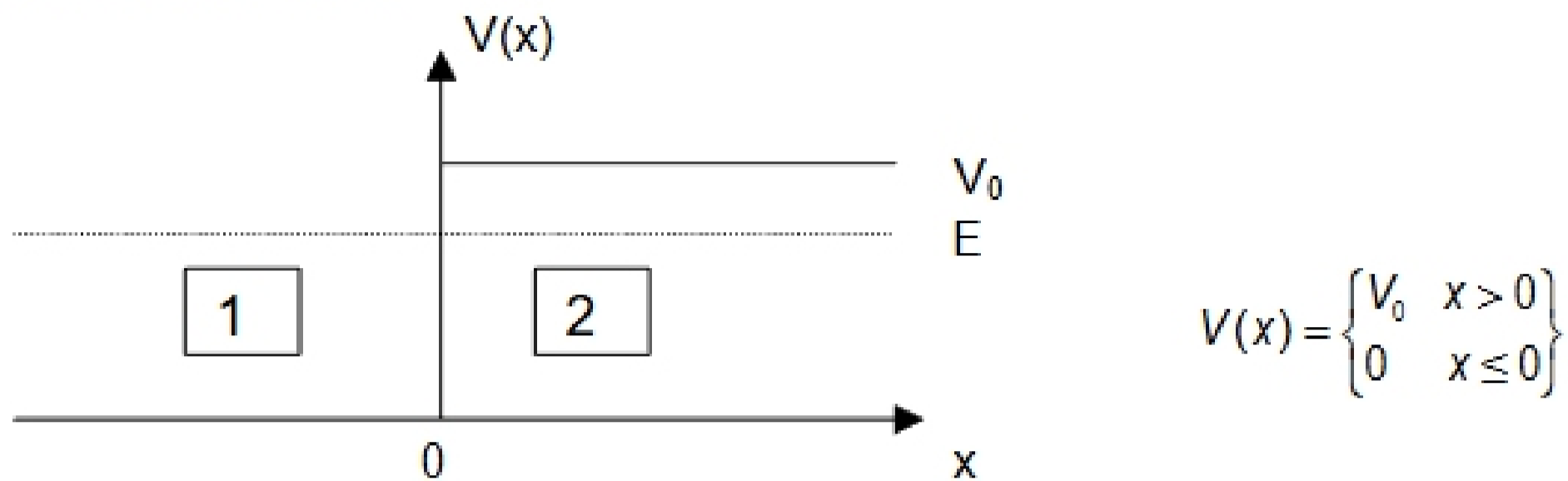
$$\int dx \sin^2 x = \frac{x}{2} - \frac{1}{4} \sin 2x \qquad \int dx x^2 \sin^2 x = \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8} \right) \sin 2x - \frac{x \cos 2x}{4}$$

$$I_n = \int_0^\infty x^n \exp(-\alpha x^2) dx \qquad I_0 = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}} \qquad I_1 = \frac{1}{2\alpha} \qquad I_2 = \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}}$$

1. [3 points] If the wavelength of an electron is known to be 1 nm, what is its kinetic energy?

2. [3 points] If the lifetime of an atom in an excited state is 1 ns (10^{-9} s), what is the intrinsic uncertainty in the energy of the photon emitted in the transition to the ground state?

3. Consider the step potential shown below. The energy E of a particle of mass m is less than the barrier height V_0 .



(a) [3 points] The solution to the time-independent Schrodinger Equation for region 1 ($x < 0$) can be written $\psi_1(x) = e^{ikx} + R e^{-ikx}$. What is the constant k ?

(b) [4 points] Solve the time-independent Schrodinger Equation to obtain the *general* solution for region 2 ($x > 0$). You should have at least one constant analogous to k in region 1 which depends on the barrier height, the particle energy, the particle mass, and \hbar .

(c) [3 points] Apply the boundary conditions at $x = \infty$ and $x = 0$ to eliminate any constants and to solve the remaining constants in terms of R .