EENS 2110	Mineralogy
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Axial Ratios, Parameters, Miller Indices

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We've now seen how crystallographic axes can be defined for the various crystal systems. Two important points to remember are that

1. The lengths of the crystallographic axes are controlled by the dimensions of the unit cell upon which the crystal is based.

2. The angles between the crystallographic axes are controlled by the shape of the unit cell.

We also noted last time that the relative lengths of the crystallographic axes control the angular relationships between crystal faces. This is true because crystal faces can only develop along lattice points. The relative lengths of the crystallographic axes are called axial ratios, our first topic of discussion.

Axial Ratios

Axial ratios are defined as the relative lengths of the crystallographic axes. They are normally taken as relative to the length of the b crystallographic axis. Thus, an axial ratio is defined as follows:

Axial Ratio = a/b : b/b : c/b

where a is the actual length of the a crystallographic axis, b, is the actual length of the b crystallographic axis, and c is the actual length of the c crystallographic axis.

- For Triclinic, Monoclinic, and Orthorhombic crystals, where the lengths of the three axes are different, this reduces to

  a/b : 1 : c/b (this is usually shortened to a : 1 : c)

- For Tetragonal crystals where the length of the a and b axes are equal, this reduces to

  1 : 1 : c/b (this is usually shorted to 1 : c)

- For Isometric crystals where the length of the a, b, and c axes are equal this becomes

  1 : 1 : 1 (this is usually shorted to 1)

- For Hexagonal crystals where there are three equal length axes (a_1, a_2, and a_3) perpendicular to the c axis this becomes:

  1 : 1 : 1: c/a (usually shortened to 1 : c)

Modern crystallographers can use x-rays to determine the size of the unit cell, and thus can determine the absolute value of the crystallographic axes. For example, the mineral quartz is hexagonal, with the following unit cell dimensions as determined by x-ray crystallography:

$$a_1 = a_2 = a_3 = 4.913Å$$

$$c = 5.405Å$$

where Å stands for Angstroms = 10^{-10} meter.

Thus the axial ratio for quartz is

$$1 : 1 : 1 : 5.405/4.913$$

or

$$1: 1 : 1 : 1.1001$$

which simply says that the c axis is 1.1001 times longer than the a axes.

For orthorhombic sulfur the unit cell dimensions as measured by x-rays are:

$$a = 10.47Å$$

$$b = 12.87Å$$

$$c = 24.39Å$$

Thus, the axial ratio for orthorhombic sulfur is:

$$10.47/12.87 : 12.87/12.87 : 24.39/12.87$$

or

$$0.813 : 1 : 1.903$$

Because crystal faces develop along lattice points, the angular relationship between faces must depend on the relative lengths of the axes. Long before x-rays were invented and absolute unit cell dimensions could be obtained, crystallographers were able to determine the axial ratios of minerals by determining the angles between crystal faces. So, for example, in 1896 the axial ratios of orthorhombic sulfur were determined to be nearly exactly the same as those reported above from x-ray measurements.

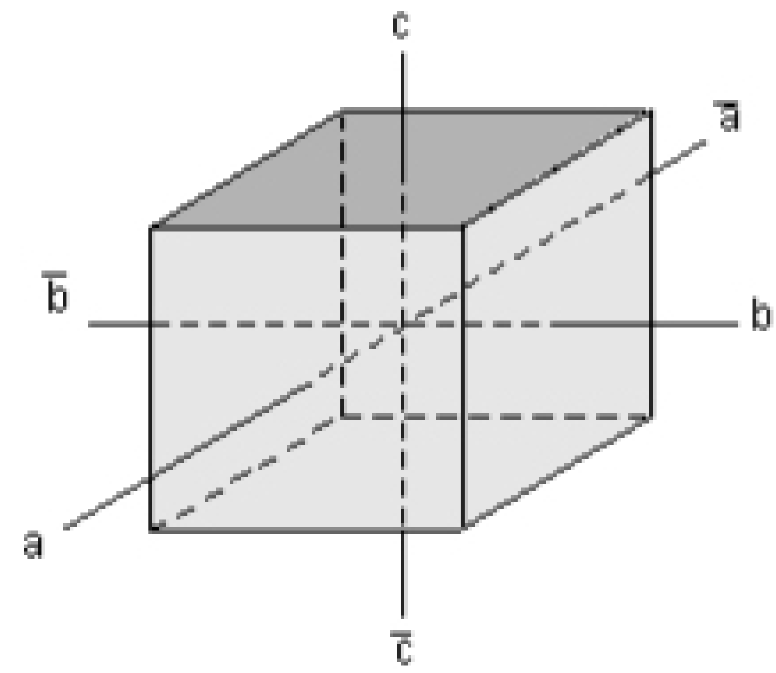
In a later lecture we will see how we can determine axial ratios from the angular relationships between faces. First, however we must determine how we can name, or index faces of crystals and define directions within crystals.

Intercepts of Crystal Faces (Weiss Parameters)

Crystal faces can be defined by their intercepts on the crystallographic axes. For non-hexagonal crystals, there are three cases.

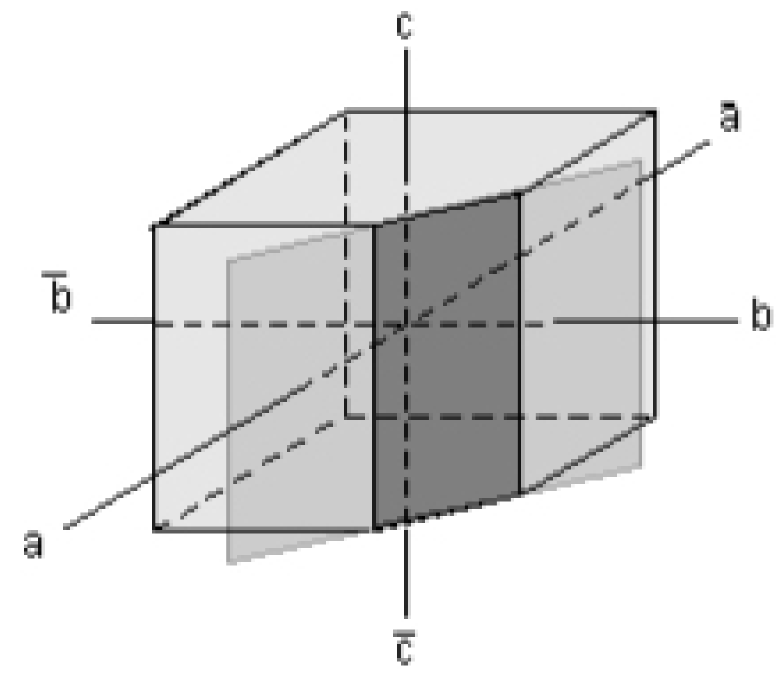
1. A crystal face intersects only one of the crystallographic axes.

   As an example the top crystal face shown here intersects the c axis but does not intersect the a or b axes. If we assume that the face intercepts the c axis at a distance of 1 unit length, then the intercepts, sometimes called Weiss Parameters, are: **∞ a, ∞ b, 1c**



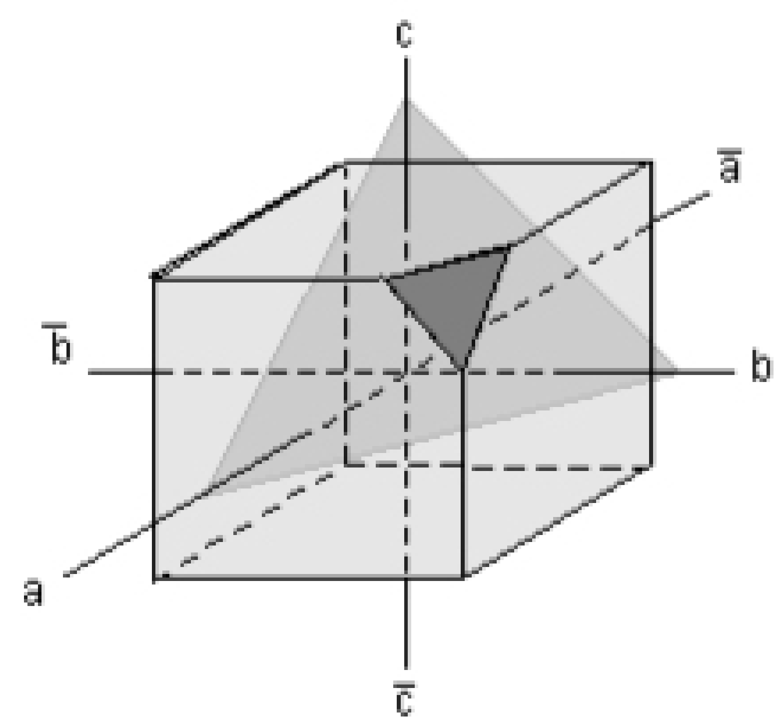
2. A crystal face intersects two of the crystallographic axes.

   As an example, the darker crystal face shown here intersects the a and b axes, but not the c axis. Assuming the face intercepts the a and c axes at 1 unit cell length on each, the parameters for this face are: **1 a, 1 b, ∞c**



3. A crystal face that intersects all 3 axes.

   In this example the darker face is assumed to intersect the a, b, and c crystallographic axes at one unit length on each. Thus, the parameters in this example would be:
   1a, 1b, 1c



Two very important points about intercepts of faces:

- **The intercepts or parameters are relative values, and do not indicate any actual cutting lengths.**
- **Since they are relative, a face can be moved parallel to itself without changing its relative intercepts or parameters**.

Because one does usually not know the dimensions of the unit cell, it is difficult to know what number to give the intercept of a face, unless one face is chosen arbitrarily to have intercepts of 1. Thus, the convention is to assign the **largest face that intersects all 3 crystallographic axes the parameters - 1a, 1b, 1c. This face is called the *unit face.***