# Max Flow, Min Cut

Minimum cut
Maximum flow
Max-flow min-cut theorem
Ford-Fulkerson augmenting path algorithm
Edmonds-Karp heuristics
Bipartite matching



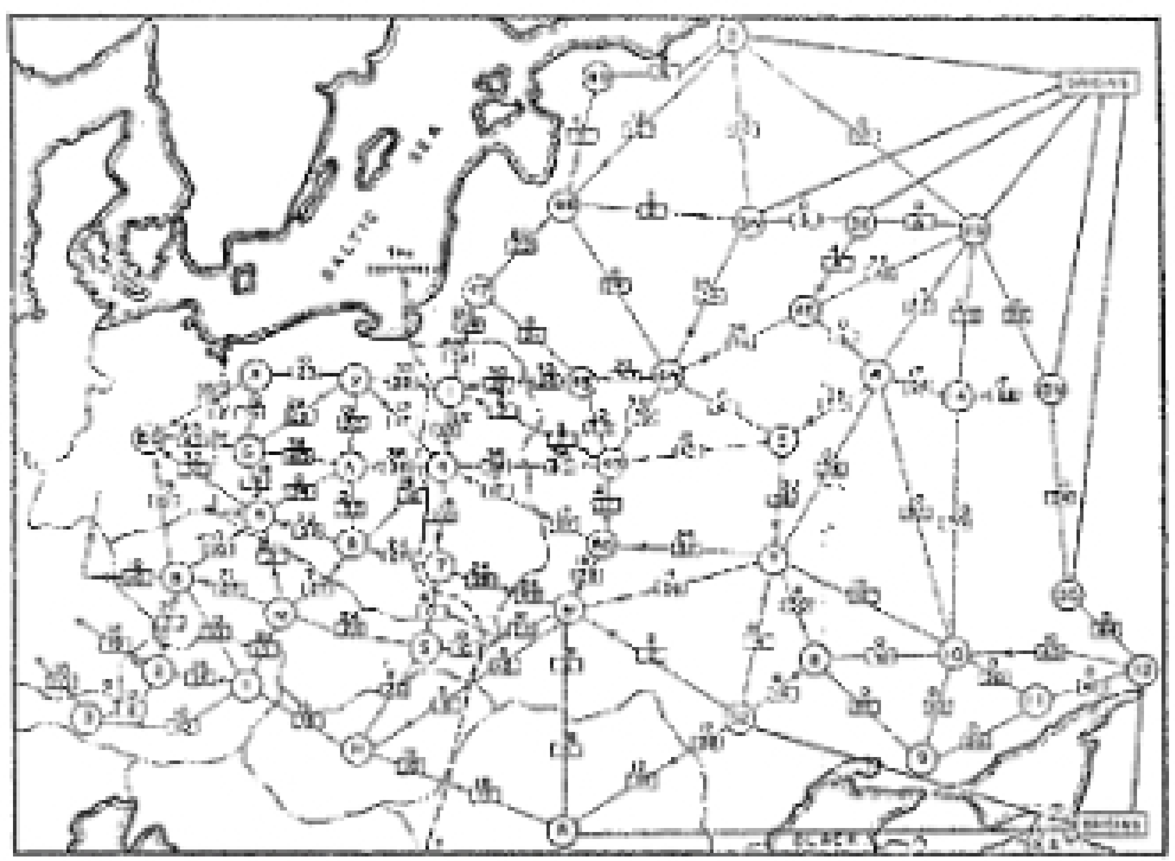
## Maximum Flow and Minimum Cut

Max flow and min cut.
- Two very rich algorithmic problems.
- Cornerstone problems in combinatorial optimization.
- Beautiful mathematical duality.

Nontrivial applications / reductions.
- Network connectivity.
- Bipartite matching.
- Data mining.
- Open-pit mining.
- Airline scheduling.
- Image processing.
- Project selection.
- Baseball elimination.
- Network reliability.
- Security of statistical data.
- Distributed computing.
- Egalitarian stable matching.
- Distributed computing.
- Many many more . . .



## Soviet Rail Network, 1955



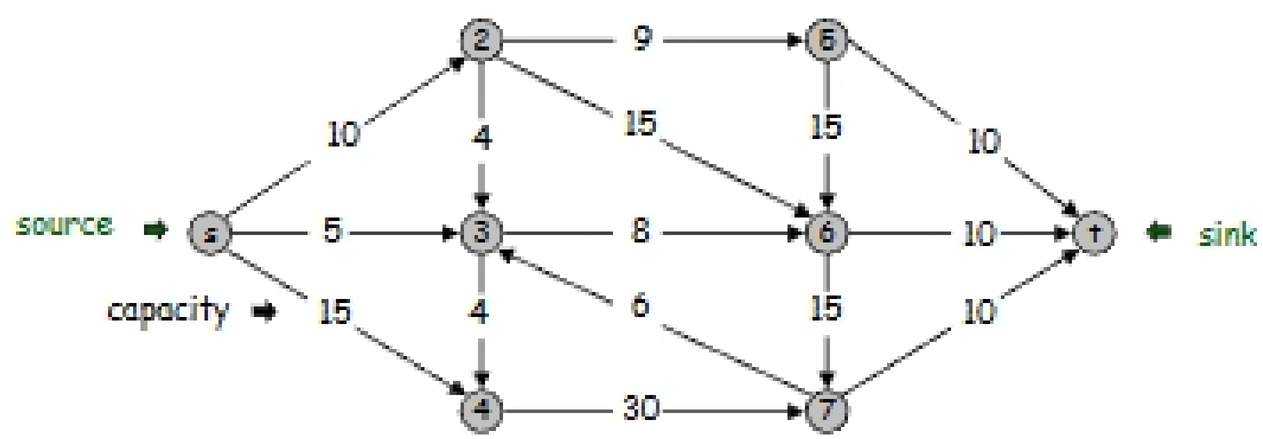
Source: *On the history of the transportation and maximum flow problems.*
Alexander Schrijver in Math Programming, 91: 3, 2002.



## Minimum Cut Problem

Network: abstraction for material FLOWING through the edges.
- Directed graph.
- Capacities on edges.
- Source node s, sink node t.

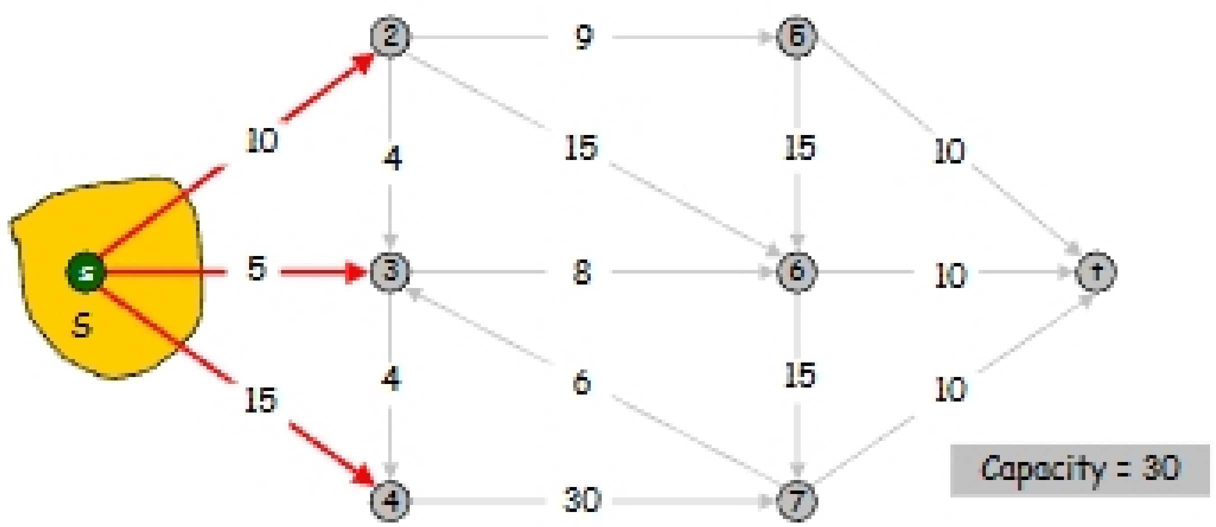
Min cut problem. Delete "best" set of edges to disconnect t from s.



## Cuts

A cut is a node partition (S, T) such that s is in S and t is in T.

- capacity(S, T) = sum of weights of edges leaving S.

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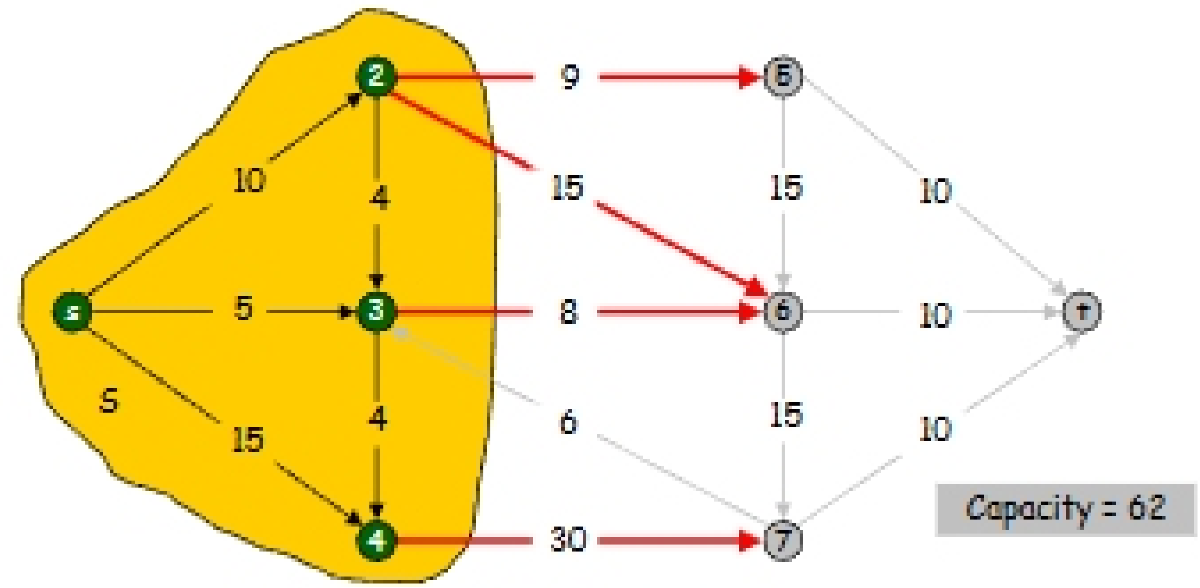


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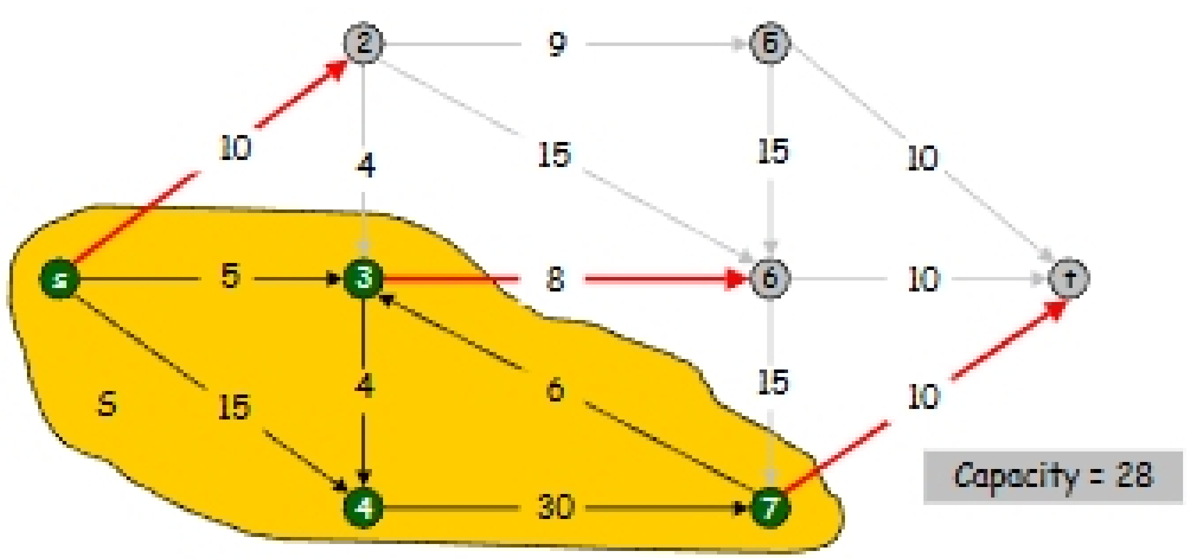
Min cut problem. Find an s-t cut of minimum capacity.




## Maximum Flow Problem

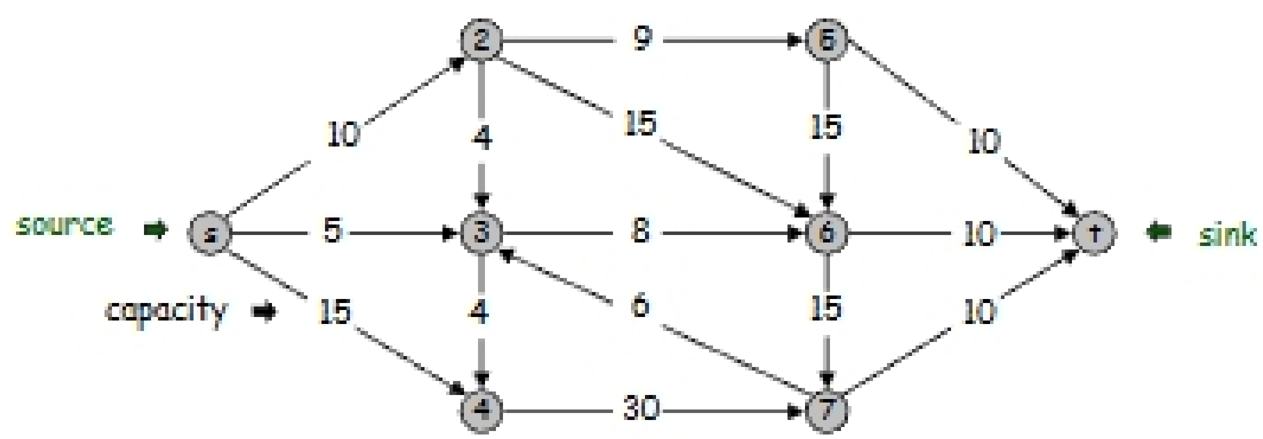
Network: abstraction for material FLOWING through the edges.

- Directed graph.
- Capacities on edges.
- Source node s, sink node t.

same input as min cut problem

Max flow problem. Assign flow to edges so as to:

- Equalize inflow and outflow at every intermediate vertex.
- Maximize flow sent from s to t.

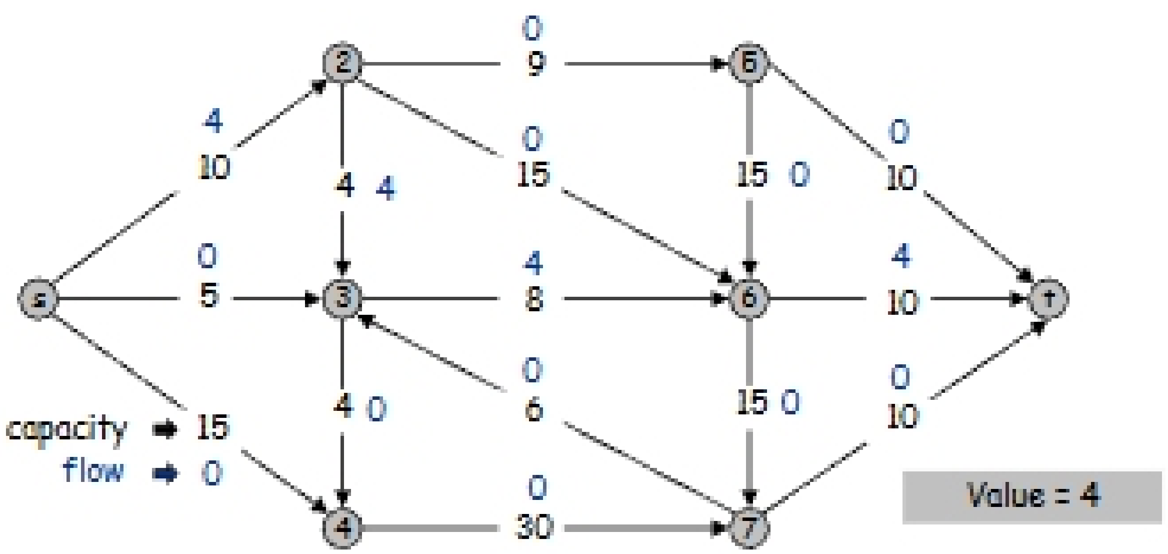


## Flows

A flow f is an assignment of weights to edges so that:

- Capacity: $0 \leq f(e) \leq u(e)$.
- Flow conservation: flow leaving v = flow entering v.

except at s or t

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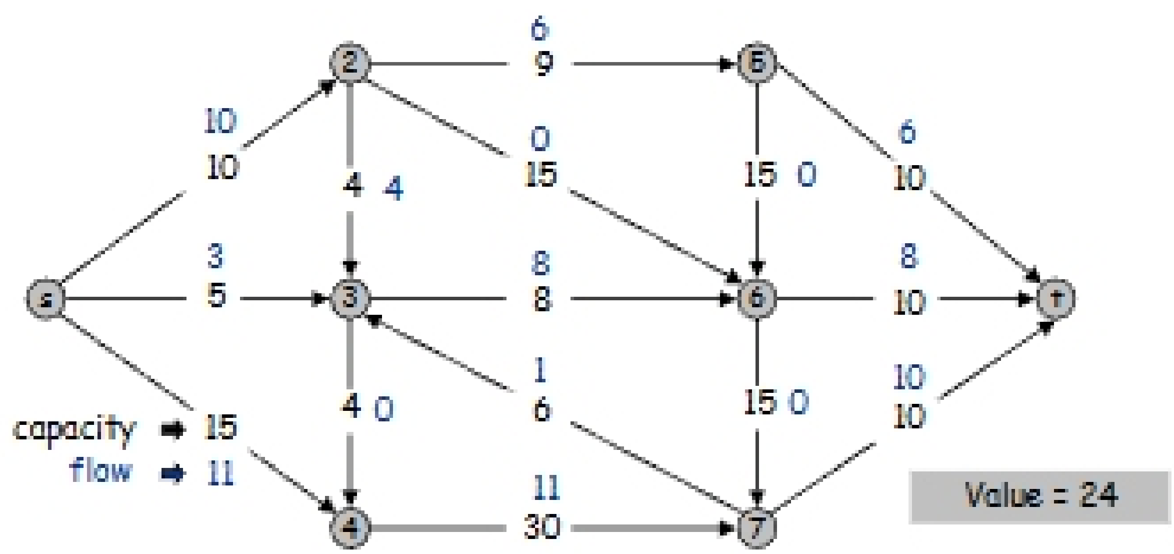
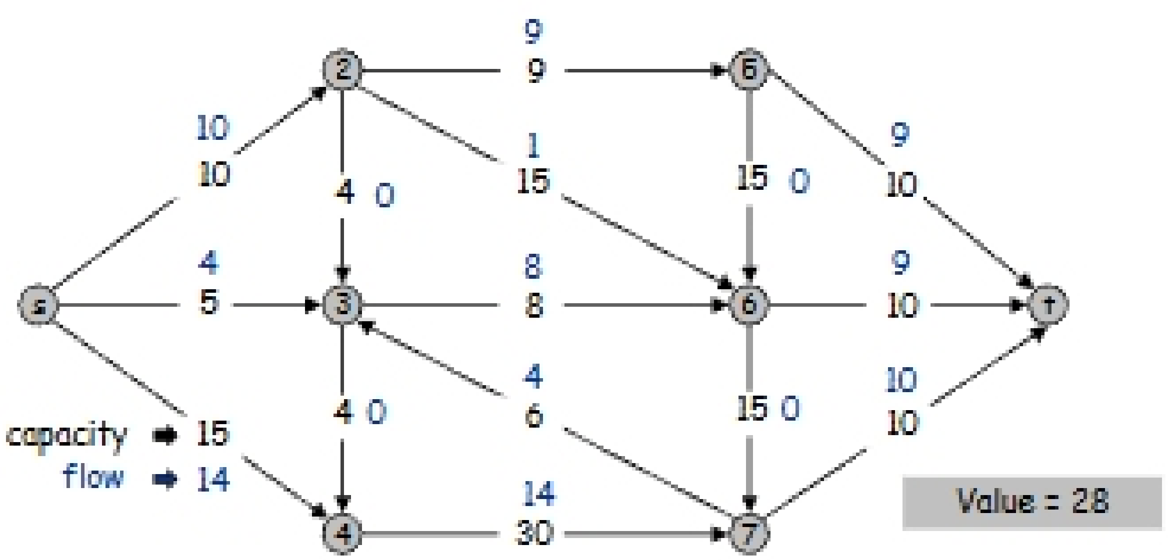



## Maximum Flow Problem

Max flow problem: find flow that maximizes net flow into sink.




## Flows and Cuts

Observation 1. Let f be a flow, and let (S, T) be any s-t cut. Then, the net flow sent across the cut is equal to the amount reaching t.