

## MAT 127: Calculus C

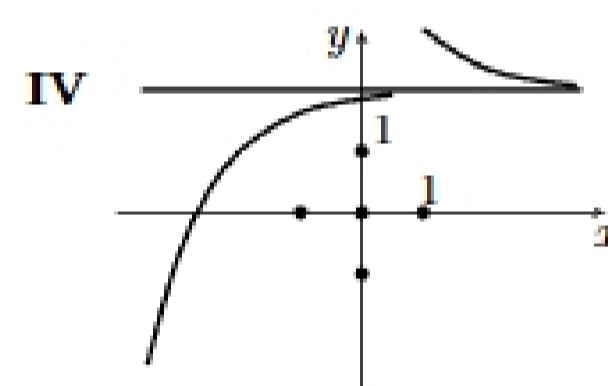
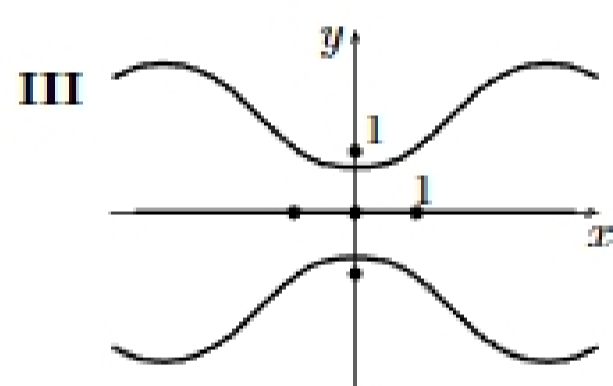
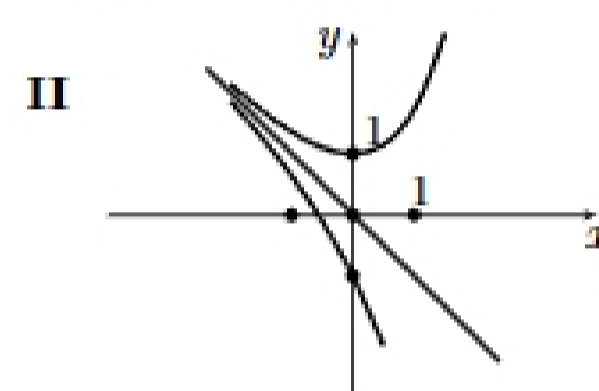
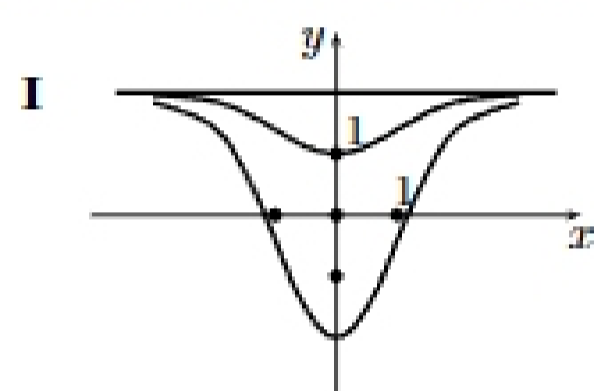
### Solutions to Mini-Quiz 1: *qualitative properties of 1st-order differential equations*

#### Problem A

Consider the four differential equations for  $y = y(x)$ :

$$(a) y' = 2 - y, \quad (b) y' = x(2 - y), \quad (c) y' = x + y - 1 \quad (d) y' = \sin x \sin y.$$

Each of the four diagrams below shows three solution curves for one of these equations:



Match each of the diagrams to the corresponding differential equation (the match is one-to-one) and explain your reasoning:

diagram	I	II	III	IV
equation	b	c	d	a

- I and IV are not (c) or (d) because the constant function  $y = 2$  is not a solution of (c) or (d)
- I is not (a) because some solution curves in I descend when  $y < 2$  (and  $x < 0$ ); IV is not (b) because one of the solution curves in IV ascends when  $y < 2$  and  $x < 0$
- III is not (a), (b), or (c) because the constant function  $y = 0$  is not a solution of (a), (b), or (c)
- II is not (a), (b), or (d) because the function  $y = -x$  is not a solution of (a), (b), or (d); also the curves in II do not have zero slope for  $y = 2$  or  $y = 0$

**Problem B:** 7.2 3-6, p506

diagram	I	II	III	IV
equation	4	6	3	5

- I is not 3 because the slopes in I depend on  $x$  (change under horizontal shifts); I is not 5 or 6 because the slopes in I are horizontal for  $y=2$
- II is not 3, 4, or 5 because the slopes in II are horizontal for  $y=0$ ; II is not 3 also because the slopes in II depend on  $x$
- III is not 4, 5, or 6 because the slopes in III do not depend on  $x$  (do not change under horizontal shifts)
- IV is not 3, 4, or 6 because the slopes are not 0 for either  $y=2$  or  $y=0$  (at least on the  $y$ -axis, where  $x=0$ )

*Remark:* The above justifications, for Problems A and B, contain 12 elimination statements: each of the 4 diagrams is shown to be incompatible with 3 of the equations. Since you know that the match is one-to-one, it is possible to fully justify the answer with just 6 elimination statements, provided they are chosen properly. For example, after you match diagram I with equation 4 in Problem B, you can forget about equation 4 when considering the remaining 3 diagrams.

**Solutions to Mini-Quiz 2:**  
*convergence/divergence of sequences/series*

*Determine whether each of the following sequences or series converges or not. In each case, clearly circle either **YES** or **NO**, but not both.*

(a) the sequence  $a_n = \frac{(-1)^{n-1}n}{n^2 + 1}$  **YES** **NO**

Since  $a_n = \frac{(-1)^n}{n} \cdot \frac{1}{1 + 1/n^2}$ , this sequence converges to 0.

(b) the sequence  $a_n = 1 + \cos(2/n)$  **YES** **NO**

Since  $\cos(2/n) \rightarrow \cos 0 = 1$ ,  $a_n \rightarrow 2$

(c) the sequence  $a_n = n \cos n$  **YES** **NO**

Since  $\cos n$  does not approach 0 as  $n \rightarrow \infty$ ,  $|a_n|$  takes arbitrarily large values

(d) the sequence  $a_n = (-1)^n \frac{n}{n+1}$  **YES** **NO**

Since  $a_n = (-1)^n \frac{1}{1+1/n}$ , the odd terms approach -1, while the even terms approach 1.

(e) the sequence  $a_n = \frac{\sin 2n}{1 + \sqrt{n}}$  **YES** **NO**

Since  $|\sin 2n| \leq 1$ , while  $\sqrt{n} \rightarrow \infty$ ,  $a_n \rightarrow 0$