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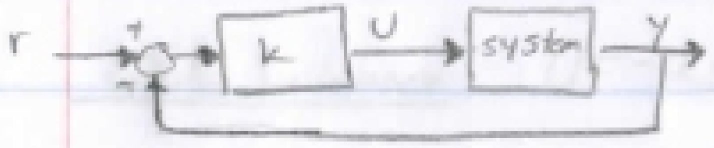
Output Feedback

$U = -ky$

input ↑ ↑ output



Goals

steady state → 1.) y becomes equal to r

transient → 2.) place the eigan values of the closed loop system @ desired locations

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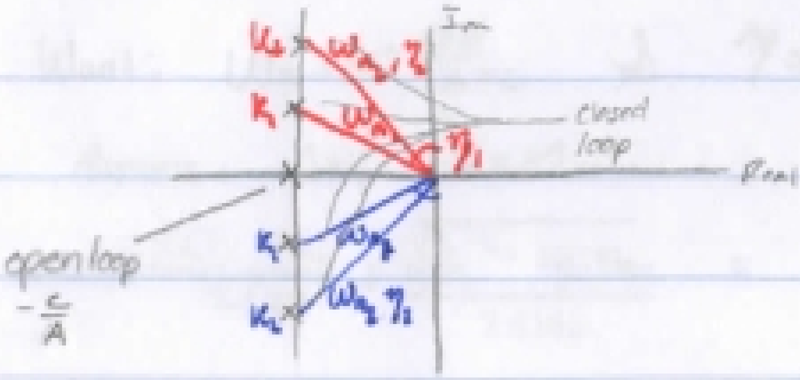
Cascade of 2 tanks

open system

$$A = \begin{bmatrix} -\frac{c}{A} & 0 \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$\lambda_{1,2} = -\frac{c}{A} \pm \frac{\sqrt{ck}}{A}$ → eigan values of closed loop

$\lambda_{1,2} = \frac{-c}{A}$ → eigan values of open loop



with $k = k_1$; ζ_1 , ω_{n_1}

$k = k_2$; $\zeta_2 < \zeta_1$; $\omega_{n_2} > \omega_{n_1}$

- with output feedback, by increasing the controll gain k, while gaining in speed by increasing ω_n, we are loosing on overshoot by decreasing ζ.
- Q: Do we have to have this compromise? Can we make ω_n & ζ to go to specific values at the same time?

$A_{cl} = A - kBC$

State vector feedback or State feedback

$U = -k\underline{x}$ → k become 1×2 matrix

↳ $A_{cl} = A - Bk$

State Feedback $U = -k\underline{x} = -\begin{bmatrix} k_1 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -k_1x_1 - k_2x_2$

$A = \begin{bmatrix} -\frac{c}{A} & 0 \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} \qquad Bk = \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} \frac{k_1}{A} & \frac{k_2}{A} \\ 0 & 0 \end{bmatrix}$

$A_{cl} = A - Bk = \begin{bmatrix} -\frac{c}{A} - \frac{k_1}{A} & -\frac{k_2}{A} \\ \frac{c}{A} & -\frac{c}{A} \end{bmatrix} \rightarrow \lambda I - A_{cl} = \begin{bmatrix} \lambda + \frac{c+k_1}{A} & \frac{k_2}{A} \\ -\frac{c}{A} & \lambda + \frac{c}{A} \end{bmatrix}$

$|\lambda I - A_{cl}| = \lambda^2 + \frac{2c+k_1}{A}\lambda + \frac{c^2}{A^2} + \frac{k_1 c}{A^2} + \frac{k_2 c}{A^2} = 0$

$$\lambda_{cl\,1,2} = \frac{\frac{-2c-k_1}{A} \pm \sqrt{\left(\frac{2c+k_1}{A}\right)^2 - 4\left(\frac{c}{A^2}\right)(c+k_1+k_2)}}{2}$$

$$\lambda_{cl\,1,2} = \frac{-2c-k_1}{2A} \pm \frac{\sqrt{k_1^2 - 4ck_2}}{2A}$$

Want: $\omega_n = 10\ \frac{rad}{sec}$ & $\eta = .7$

Assume: $c = 1$ $A = 10m^2$

$$\frac{-2(1) - k_1}{2(10)} \pm \frac{\sqrt{k_1^2 - 4(1)k_2}}{2(10)} = \frac{-2-k_1}{20} \pm \frac{\sqrt{k_1^2 - 4k_2}}{20}$$

$\lambda_{1,2} = -\eta\omega_n \pm j\omega_n\sqrt{1-\eta^2} \rightarrow -(.7)(10) \pm j(10)\sqrt{1-(.7)^2}$

$-7 \pm j(10)\sqrt{.51}$

$\frac{-2-k_1}{20} = 7 \rightarrow k_1 = 138$

$\sqrt{\frac{(138)^2 - 4k_2}{20}} = (10)\sqrt{.7-1} \rightarrow k_2 = 9861$

$U = -k(x_1 + x_2)$

$C = [1 \;\; 1]$

$\hookrightarrow y = x_1 + x_2$

$U = -ky = -k(x_1 + x_2)$

$A_{cl} = A - kBC$

$$kBC = \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} [1 \;\; 1] = \begin{bmatrix} \frac{k}{A} & \frac{k}{A} \\ 0 & 0 \end{bmatrix}$$

$$A_{cl} = \begin{bmatrix} \frac{c-k}{A} & -\frac{k}{A} \\ \frac{-c}{A} & \frac{c}{A} \end{bmatrix}$$

$$|\lambda I - A_{cl}| = \lambda^2 + \frac{2c-k}{A}\lambda + \frac{c^2 - ck}{A} - \frac{ck}{A} = 0$$

$$\lambda_{1,2} = \frac{1}{2}\left[\frac{k-2c}{A}\right] \pm \sqrt{\left(\frac{2c-k}{A}\right)^2 - 4\left(\frac{c^2 - 2ck}{A}\right)}$$

Controllable/Reachable: if with state feedback control we can place the eiganvalues anywhere on the complex plane.

Reachability matrix: $W_r = [B \; AB \; A^2B \dots A^{n-1}B]$

if $|W_r| \neq 0 \Rightarrow$ the system is reachable/controlable

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$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad x = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$ $\quad T = 1.2$ seconds