

Math 19. Problem Set # 1 Solutions
Ex. 1, 2, 4 (a, b, c, d), 5, page 43-44

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Ex. 1

a)

$$\begin{aligned}\frac{dy}{dt} &= 5y \\ \int \frac{dy}{y} &= \int 5dt \\ \ln y &= 5t + C \\ y &= e^{5t+C} = C \cdot e^{5t}\end{aligned}$$

b)

$$\begin{aligned}\frac{dy}{dt} &= -3y \\ \int \frac{dy}{y} &= \int (-3)dt \\ \ln y &= -3t + C \\ y &= e^{-3t+C} = C \cdot e^{-3t}\end{aligned}$$

c)

$$\begin{aligned}\frac{dy}{dt} &= 12y \\ \int \frac{dy}{y} &= \int 12dt \\ \ln y &= 12t + C \\ y &= e^{12t+C} = C \cdot e^{12t}\end{aligned}$$

d)

$$\begin{aligned}\frac{dy}{dt} &= -1.5y \\ \int \frac{dy}{y} &= \int (-1.5)dt \\ \ln y &= -1.5t + C \\ y &= e^{-1.5t+C} = C \cdot e^{-1.5t}\end{aligned}$$

Where C is a constant.

You can also just use the formula for the exponential growth equation, going directly from $\frac{dp}{dt} = ap$ to $p(t) = p(0) \cdot e^{at}$.

Thus, you can write:

a)

$$\frac{dy}{dt} = 5y$$
$$y(t) = y(0) \cdot e^{5t}$$

b)

$$\frac{dy}{dt} = -3y$$
$$y(t) = y(0) \cdot e^{-3t}$$

c)

$$\frac{dy}{dt} = 12y$$
$$y(t) = y(0) \cdot e^{12t}$$

d)

$$\frac{dy}{dt} = -1.5y$$
$$y(t) = y(0) \cdot e^{-1.5t}$$

Ex. 2

a)

$$y(0) = 1$$

1a) $y(t) = y(0) \cdot e^{5t}$
 $y(t) = e^{5t}$

1b) $y(t) = y(0) \cdot e^{-3t}$
 $y(t) = e^{-3t}$

1c) $y(t) = y(0) \cdot e^{12t}$
 $y(t) = e^{12t}$

1d) $y(t) = y(0) \cdot e^{-1.5t}$
 $y(t) = e^{-1.5t}$

b)

$$y(1) = 1$$

1a) $y(t) = y(0) \cdot e^{5t}$
 $1 = y(0) \cdot e^{5 \cdot 1}$
 $y(0) = e^{-5}$
 $y(t) = e^{-5 + 5t}$

$$\begin{aligned}
1b) \quad y(t) &= y(0) \cdot e^{-3t} \\
1 &= y(0) \cdot e^{-3 \cdot 1} \\
y(0) &= e^3 \\
y(t) &= e^{3-3t}
\end{aligned}$$

$$\begin{aligned}
1c) \quad y(t) &= y(0) \cdot e^{12t} \\
1 &= y(0) \cdot e^{12 \cdot 1} \\
y(0) &= e^{-12} \\
y(t) &= e^{-12+12t}
\end{aligned}$$

$$\begin{aligned}
1d) \quad y(t) &= y(0) \cdot e^{-1.5t} \\
1 &= y(0) \cdot e^{-1.5 \cdot 1} \\
y(0) &= e^{1.5} \\
y(t) &= e^{1.5-1.5t}
\end{aligned}$$

c)

$$y(-1) = 1$$

$$\begin{aligned}
1a) \quad y(t) &= y(0) \cdot e^{5t} \\
1 &= y(0) \cdot e^{-5 \cdot 1} \\
y(0) &= e^5 \\
y(t) &= e^{5+5t}
\end{aligned}$$

$$\begin{aligned}
1b) \quad y(t) &= y(0) \cdot e^{-3t} \\
1 &= y(0) \cdot e^{3 \cdot 1} \\
y(0) &= e^{-3} \\
y(t) &= e^{-3-3t}
\end{aligned}$$

$$\begin{aligned}
1c) \quad y(t) &= y(0) \cdot e^{12t} \\
1 &= y(0) \cdot e^{-12 \cdot 1} \\
y(0) &= e^{12} \\
y(t) &= e^{12+12t}
\end{aligned}$$

$$\begin{aligned}
1d) \quad y(t) &= y(0) \cdot e^{-1.5t} \\
1 &= y(0) \cdot e^{1.5 \cdot 1} \\
y(0) &= e^{-1.5} \\
y(t) &= e^{-1.5-1.5t}
\end{aligned}$$