Lecture 36: Modeling Computing

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How convincing is our Halting Problem proof?

```
(define (contradict-halts x)
  (if (halts? contradict-halts)
      (loop-forever)
      #t))
```

contradicts-halts cannot exist. Everything we used to make it except halts? does exist, therefore halts? cannot exist.

This "proof" assumes Scheme exists and is consistent!

DrScheme

Is DrScheme a proof that Scheme exists?

```
(define (make-huge n)
    (if (= n 0) null
        (cons (make-huge (- n 1))
              (make-huge (- n 1)))))
(make-huge 10000)
```

Scheme/Charme/Python/etc. all fail to evaluate some program!

Solutions

- Option 1: Prove "Scheme" does exist
  - Show that we could implement all the evaluation rules (if we had "Python", our Charme interpreter would be a good start, but we don't have "Python")
- Option 2: Find a simpler computing model
  - Define it precisely
  - Show that "contradict-halts" can be defined in this model

Modeling Computation

- For a more convincing proof, we need a more precise (but simple) model of what a computer can do
- Another reason we need a model:

  Does complexity really make sense without this? (how do we know what a "step" is? are they the same for all computers?)

What is a model?

$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

How should we model a Computer?

Turing invented the model we'll use today in 1936. What "computer" was he modeling?

"Computers" before WWII

Modeling Computers

- Input
  - Without it, we can't describe a problem
- Output
  - Without it, we can't get an answer
- Processing
  - Need some way of getting from the input to the output
- Memory
  - Need to keep track of what we are doing

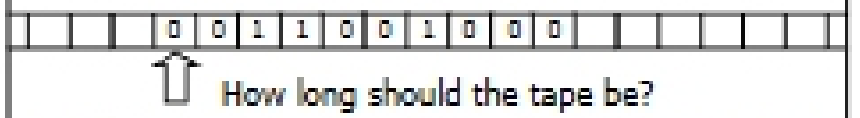
Modeling Input

Turing's "Computer"

"Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book."
Alan Turing, *On computable numbers, with an application to the Entscheidungsproblem*, 1936

Simplest Input

- Non-interactive: like punch cards and paper tape
- One-dimensional: just a single tape of values, pointer to one square on tape



How long should the tape be?

Infinitely long! We are *modeling* a computer, not building one. Our model should not have silly practical limitations (like a real computer does).

Modeling Output

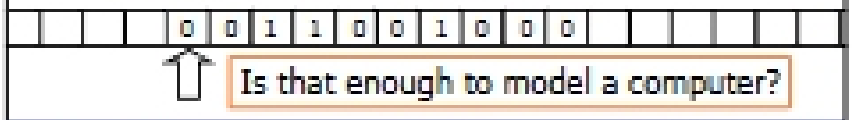
- Blinking lights are cool, but hard to model
- Output is what is written on the tape at the end of a computation

Modeling Processing (Brains)

"For the present I shall only say that the justification lies in the fact that the human memory is necessarily limited."
Alan Turing

Modeling Processing

- Evaluation Rules
  - Given an input on our tape, how do we evaluate to produce the output
- What do we need:
  - Read what is on the tape at the current square
  - Move the tape one square in either direction
  - Write into the current square

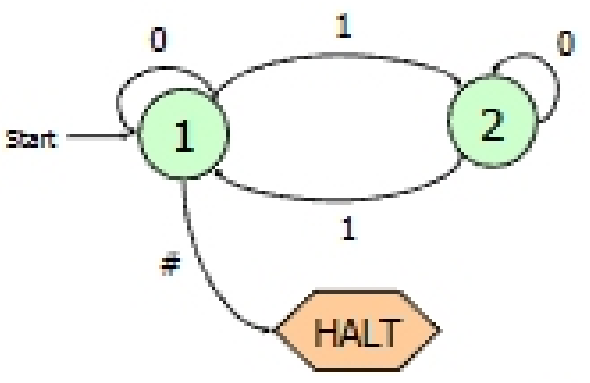


Is that enough to model a computer?

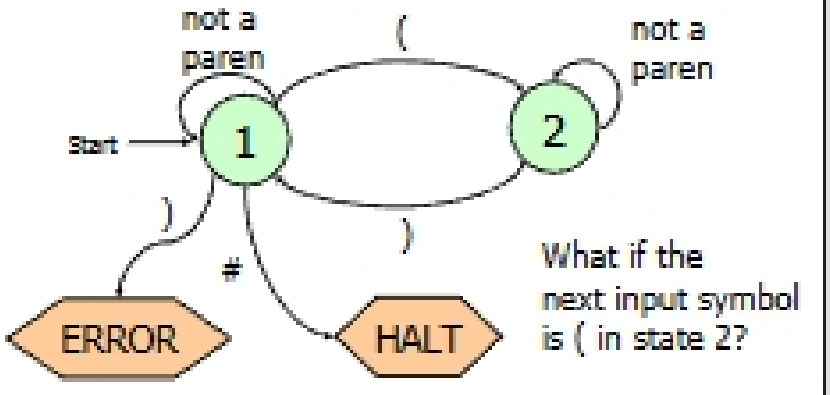
Modeling Processing

- Read, write and move is not enough
- We also need to keep track of what we are doing:
  - How do we know whether to read, write or move at each step?
  - How do we know when we're done?
- What do we need for this?

Finite State Machines



Hmmm…maybe we don't need those infinite tapes after all?



What if the next input symbol is (in state 2?