Chapter 2

System Modeling

2.1 Introduction

In this chapter we introduce the notion of a dynamical system and describe how to model system systems. Roughly speaking, a dynamical system is one in which the effects of actions do not occur immediately. For example, the velocity of a car does not change immediately when the gas pedal is pushed nor does the temperature in a room rise instantaneously when an air conditioner is switched on. Similarly, a headache does not vanish right after an aspirin is taken, requiring time to take effect. In business systems, increased funding for a development project does not increase revenues in the short term, although it may do so in the long term (if it was a good investment). All of these are examples of dynamical systems, in which the behavior of the system evolves with time.

Modeling is the method by which we deseribe a dynamical system in a precise mathematical form, for the purpose of analysis and simulation. A model of a system is a *representation* of the system dynamics and it is used to answer questions about that system. The model we choose depends on the questions that we wish to answer, and so there may be multiple models for a single physical system, with different levels of fidelity depending on the phenomena of interest. In this chapter we provide an introduction to the concept of modeling, and provide some basic material on two specific methods that are commonly used in feedback and control systems: differential equations and different equations.

2.2 Two Views on Dynamics

Dynamical systems can be viewed from two different ways: the internal view or the external view. The internal view which attempts to describe the internal workings of the system originates from classical mechanics. The prototype problem was the problem to describe the motion of the planets. For this problem it was natural to give a complete characterization of the motion of all planets. This involves careful analysis of the effects of gravitational pull and the relative positions of the planets in a system.

The other view on dynamics originated in electrical engineering. The prototype problem was to describe electronic amplifiers. It was natural to view an amplifier as a device that transforms input voltages to output voltages and disregard the internal detail of the amplifier. This resulted in the input-output view of systems. The two different views have been amalgamated in control theory. Models based on the internal view are called internal descriptions, state models, or white box models. The external view is associated with names such as external descriptions, input-output models or black box models. In this book we will mostly use the words state models and input-output models.

The Heritage of Mechanics

Dynamics originated in the attempts to describe planetary motion. The basis was detailed observations of the planets by Tycho Brahe and the results of Kepler who found empirically that the orbits could be well described by ellipses. Newton embarked on an ambitious program to try to explain why the planets move in ellipses and he found that the motion could be explained by his law of gravitation and the formula that force equals mass times acceleration. In the process he also invented calculus and differential equations. Newtons results was the first example of the idea of reductionism, i.e. that seemingly complicated natural phenomena can be explained by simple physical laws. This became the paradigm of natural science for many centuries.

One of the triumphs of Newton's mechanics was the observation that the motion of the planets could be predicted based on the current positions and velocities of all planets. It was not necessary to know the past motion. The *state* of a dynamical system is a collection of variables that characterize the motion of a system completely for the purpose of predicting future motion. For a system of planets the state is simply the positions and the velocities of the planets. A mathematical model simply gives the rate of change of the

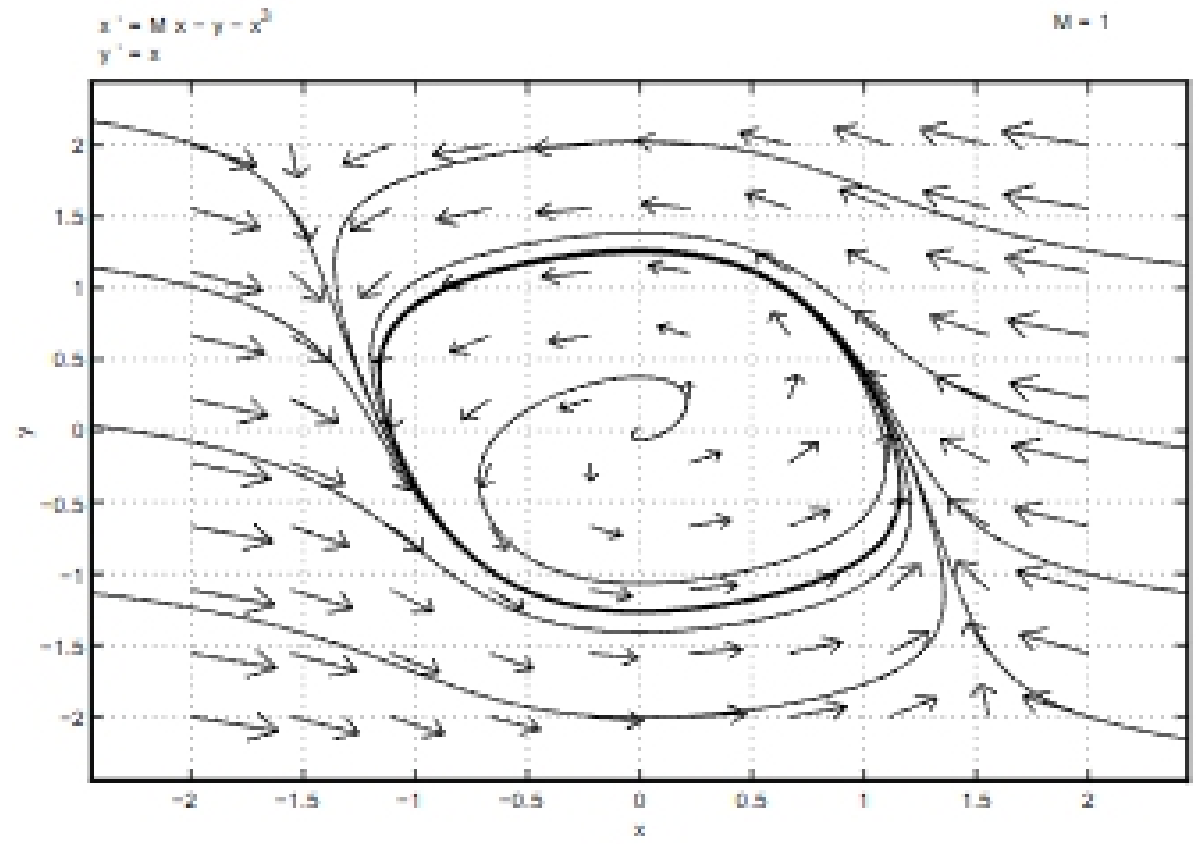

Figure 2.1: Illustration of a state model. A state model gives the rate of change of the state as a function of the state. The velocity of the state are denoted by arrows.

state as a function of the state itself, i.e. a differential equation.

$$\frac{dx}{dt} = f(x) \tag{2.1}$$

This is illustrated in Figure 2.1 for a system with two state variables. The particular system represented in the figure is the van der Pol equation:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1 - x_1^3 - x_2\\ \frac{dx_2}{dt} &= x_1,\end{aligned}$$

which is a model of an electronic oscillator. The model (2.1) gives the velocity of the state vector for each value of the state. These are represented by the arrows in the figure. The figure gives a strong intuitive representation of the equation as a vector field or a flow. Systems of second order can be represented in this way. It is unfortunately difficult to visualize equations of higher order in this way.

The ideas of dynamics and state have had a profound influence on philosophy where it inspired the idea of predestination. If the state of a natural system is known at some time, its future development is complete determined. The vital development of dynamics has continued in the 20th century. One of the interesting outcomes is chaos theory. It was discovered that