CDS 101: Lecture 2.1
System Modeling

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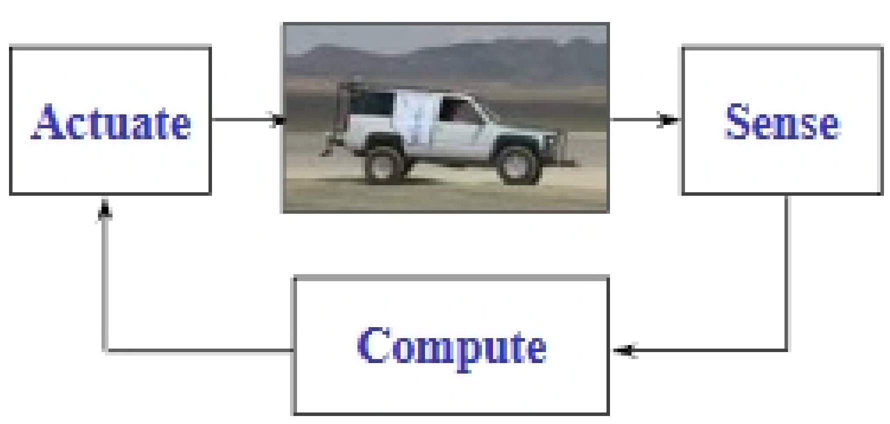
Goals:
- Define what a model is and its use in answering questions about a system
- Introduce the concepts of state, dynamics, inputs and outputs
- Provide examples of common modeling techniques: differential equations, difference equations, finite state automata

Reading:
- Åström and Murray, *Analysis and Design of Feedback Systems,* Ch 2
- Advanced: Lewis, *A Mathematical Approach to Classical Control*, Ch 1

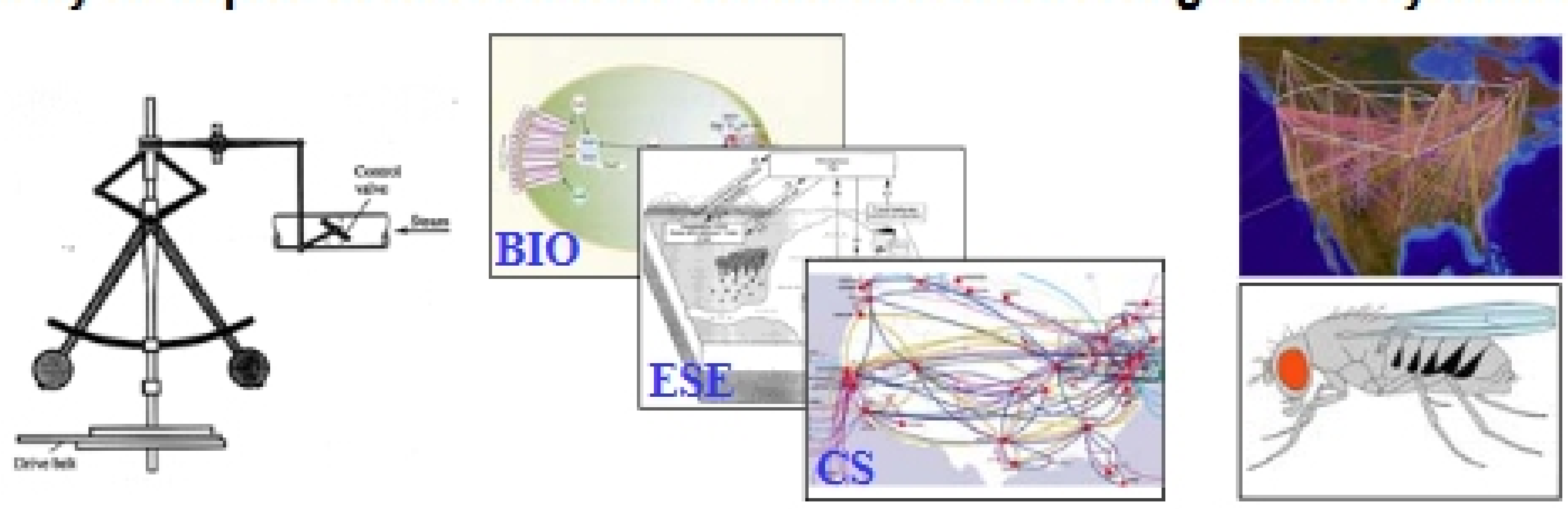
Review from last week



Control =
Sensing + Computation + Actuation

Feedback Principles
- Robustness to Uncertainty
- Design of Dynamics

Many examples of feedback and control in natural & engineered systems:



Model-Based Analysis of Feedback Systems

Analysis and design based on *models*

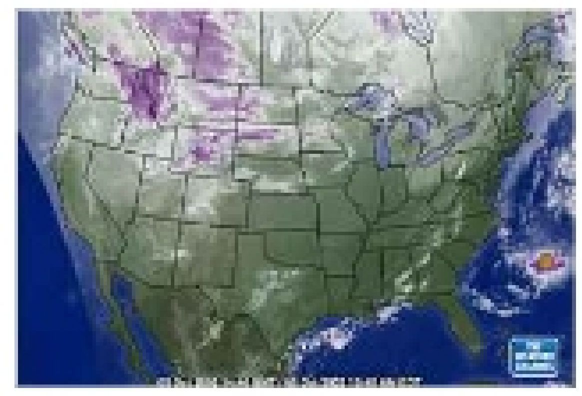
- A model provides a *prediction* of how the system will behave
- Feedback can give counter-intuitive behavior; models help sort out what is going on
- For control design, models don't have to be exact: *feedback* provides robustness

Control-oriented models: *inputs* and *outputs*

The model you use depends on the questions you want to answer

- A single system may have many models
- Time and spatial scale must be chosen to suit the questions you want to answer
- Formulate questions *before* building a model

Weather Forecasting



- Question 1: how much will it rain tomorrow?
- Question 2: will it rain in the next 5-10 days?
- Question 3: will we have a drought next summer?

Different questions ⇒ different models

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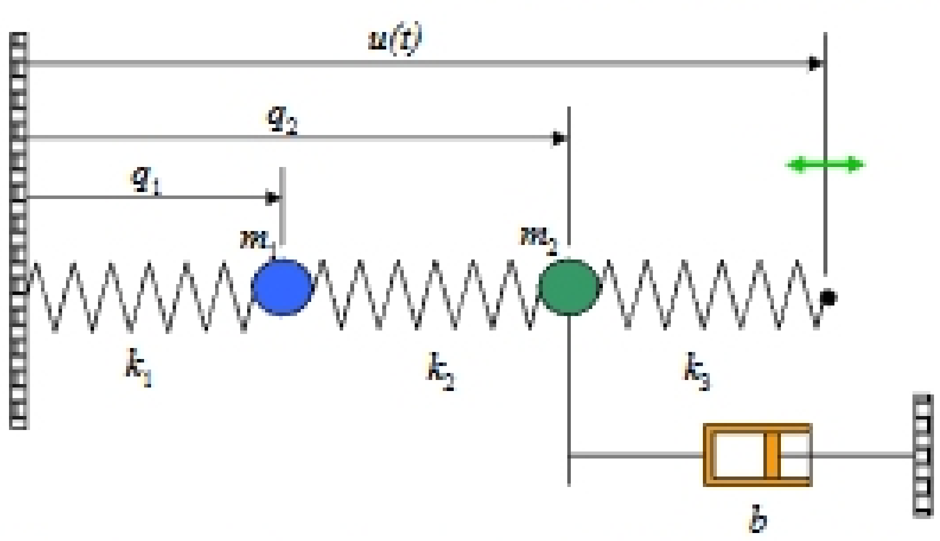
Example #1: Spring Mass System



Applications

- Flexible structures (many apps)
- Suspension systems (eg, "Bob")
- Molecular and quantum dynamics

Questions we want to answer

- How much do masses move as a function of the forcing frequency?
- What happens if I change the values of the masses?
- Will Bob fly into the air if I take that hill at 25 mph?

Modeling assumptions

- Mass, spring, and damper constants are fixed and known
- Springs satisfy Hooke's law
- Damper is (linear) viscous force, proportional to velocity

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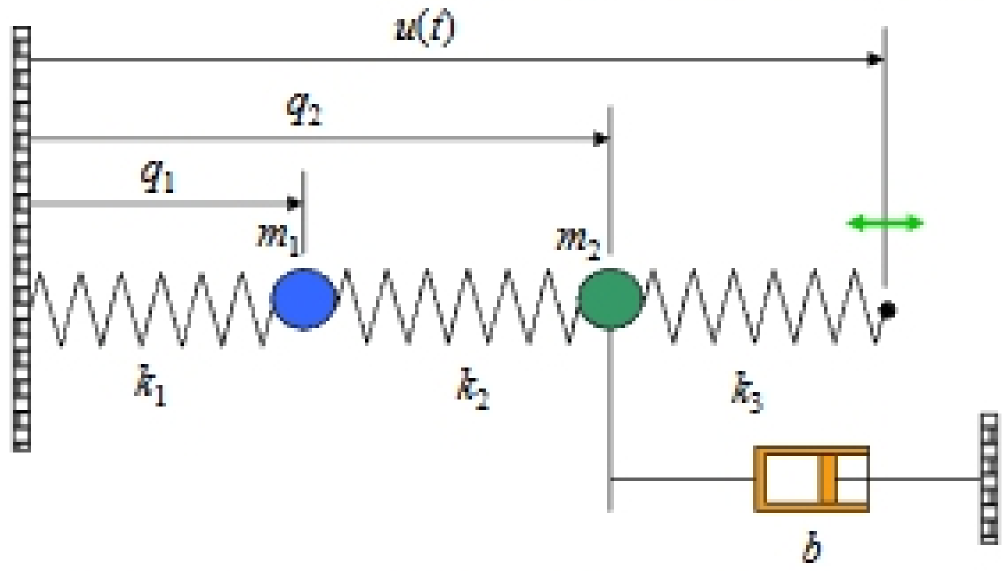
Modeling a Spring Mass System



Model: rigid body physics (Ph 1)

- Sum of forces = mass * acceleration
- Hooke's law: $F = k(x - x_{rest})$
- Viscous friction: $F = b\,v$

$$
\begin{aligned}
m_1\ddot{q}_1 &= k_2(q_2 - q_1) - k_1 q_1 \\
m_2\ddot{q}_2 &= k_3(u - q_2) - k_2(q_2 - q_1) - b\dot{q}_2
\end{aligned}
$$

Converting models to state space form

- Construct a *vector* of the variables that are required to specify the evolution of the system
- Write dynamics as a *system* of first order differential equations:

$$\frac{dx}{dt} = f(x,u) \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^p$$
$$y = h(x) \qquad y \in \mathbb{R}^q$$

$$
\frac{d}{dt}\begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \frac{k_2}{m}(q_2 - q_1) - \frac{k_1}{m} q_1 \\ \frac{k_3}{m}(u - q_2) - \frac{k_2}{m}(q_2 - q_1) - \frac{b}{m}\dot{q}_2 \end{bmatrix}
$$

$$y = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$ "State space form"

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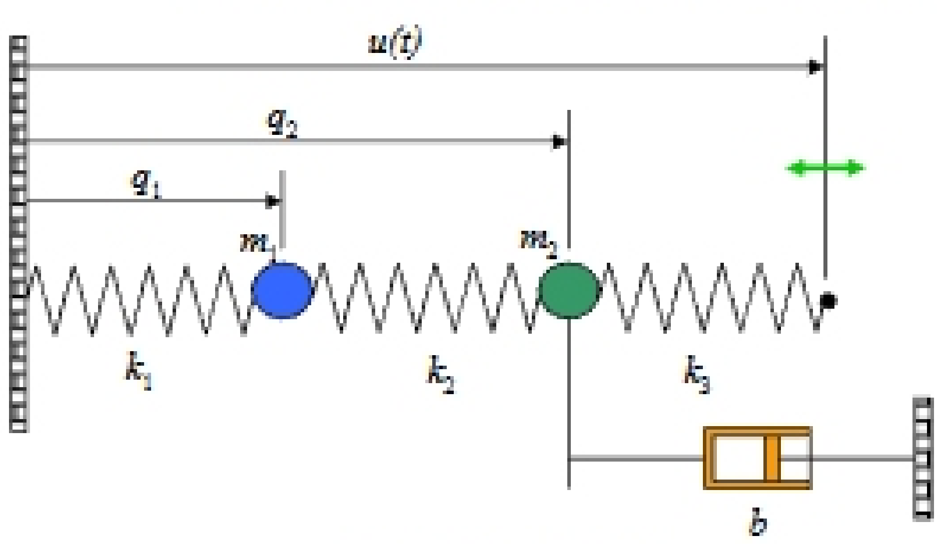
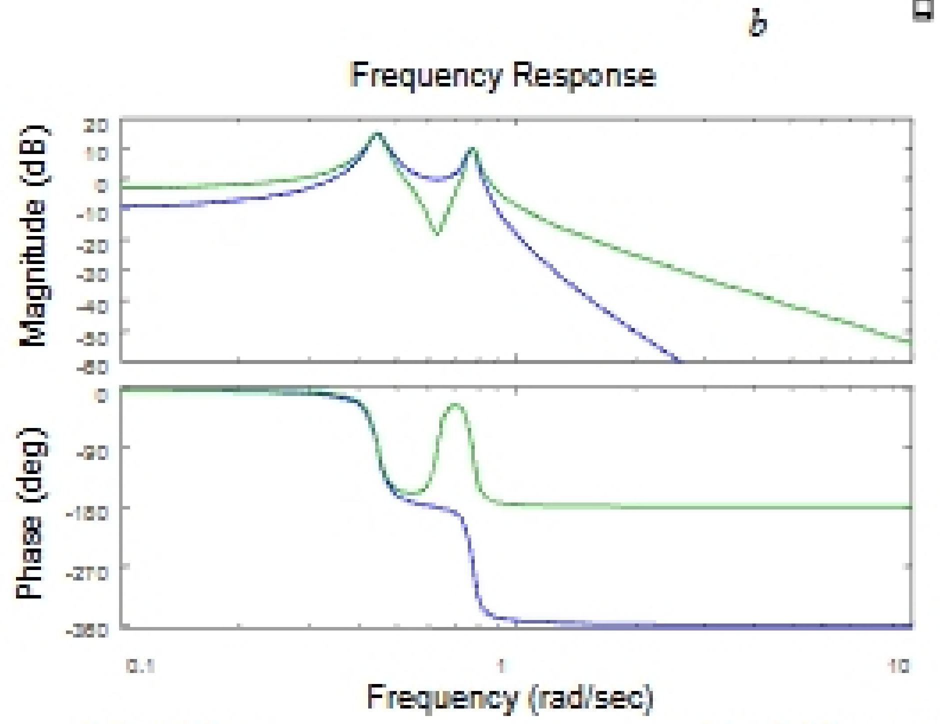
Frequency Response for a Mass Spring System





Steady state frequency response

- Force the system with a sinusoid
- Plot the "steady state" response, after transients have died out
- Plot relative magnitude and phase of output versus input (more later)

Matlab simulation (see handout)

```
function dydt = f(t, y, ...)
u = 0.00315*cos(omega*t);
dydt = [
  y(3);
  y(4);
  -(k1+k2)/m1*y(1) + k2/m1*y(2);
  k2/m2*y(1) - (k2+k3)/m2*y(2)
      - b/m2*y(4) + k3/m2*u ];

t,y] = ode45(dydt,tspan,y0,[], k1,
k2, k3, m1, m2, b, omega);
```

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