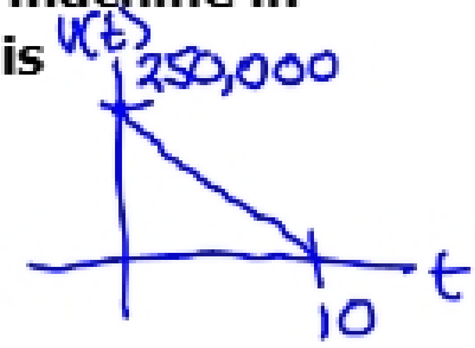
# SECTION 1.3

## Linear Functions and Math Models

**Simple Depreciation:**

**Example 1: In 2000, the B&C Company installed a new machine in one of its factories at a cost of $250,000. The machine is depreciated linearly over 10 years with no scrap value.**




t = time V(t) = value

**a. Find the rate of depreciation for this machine.**

(0, 250,000) (10, 0)

$m = \frac{0-250{,}000}{10-0} = -25{,}000$

Rate of depreciation $25,000.00

**b. Find an expression for the machine's book value in the t-th year of use (0≤t≤10).**

$y = mt + b$ $m = -25{,}000$ $b = 250{,}000$

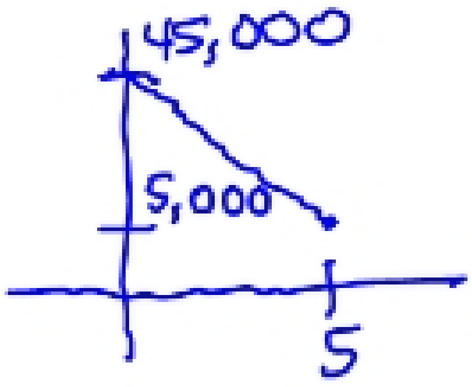
$V(t) = -25{,}000t + 250{,}000$

**c. If the machine was purchased in 2001, find the value of the machine in 2006.** 2006 − 2001 = 5

$V(5) = -25{,}000(5) + 250{,}000 = \$125{,}000.00$

**Example 2: A company's car has an original value of $45, 000 and will be depreciated linearly over 5 years with scrap value of $5,000.**

**a. Find the expression giving the book value of the car at the end of year t (0≤t≤5).** (0, 45,000) (5, 5,000)




$m = \frac{5{,}000-45{,}000}{5-0} = -8{,}000$

$V(t) = -8{,}000t + 45{,}000$

**b. If the car was purchased in 2005 find the value in 2008?**

$V(t) = -8{,}000t + 45{,}000$

$2008 - 2005 = 3$ years

$V(3) = -8{,}000(3) + 45{,}000 = \$21{,}000.00$

**Linear Cost, Revenue and Profit Functions:**

**If *x* is the number of units of a product manufactured or sold at a firm then,**

**The cost function, C(x), is the total cost of manufacturing x units of the product.**

**The revenue function, R(x), is the total revenue realized from the sale of x units of the product.**

**The profit function, P(x), is the total profit realized from the manufacturing and sale of the x units of product.**

**Fixed costs are the costs that remain regardless of the company's activity.**

**Examples: building fees (rent or mortgage), executive salaries**

**Variable costs are costs that vary with the production or sales.**

**Examples: wages of production staff, raw materials**

**Formulas:**
**Suppose a firm has fixed cost of *F* dollars, production cost of *c* dollars per unit and selling price of *s* dollars per unit then**

$C(x) = cx + F$ cost function

$R(x) = sx$ revenue function

$P(x) = R(x) - C(x) = sx - (cx + F) = sx - cx - F$

**Where *x* is the number of units of the commodity produced and sold.**

**Example 3: A manufacturer has a monthly fixed cost of $100,000 and a production cost of $14 for each unit produced. The product sells for $20 per unit.**

**a. What is the cost function?** $C(x) = 14x + 100{,}000$

**b. What is the revenue function?** $R(x) = 20x$

**c. What is the profit function?**

$P(x) = R(x) - C(x)$

$P(x) = 20x - (14x + 100{,}000)$

$P(x) = 20x - 14x - 100{,}000$

$P(x) = 6x - 100{,}000$

**d. Compute the profit (loss) corresponding to production levels of 12,000 and 20,000.**

$P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$ Loss

$P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$ profit

**e. How many units must the company produce and sell if they wish to make a profit of $50,000?**

$P(x) = 6x - 100{,}000$

$50{,}000 = 6x - 100{,}000$

$\frac{6x}{6} = \frac{150{,}000}{6}$

$x = 25{,}000$ units