2-D Lin. Systems

Friday, November 9, 2012 10:00 AM

(Phase Plane)

Def: $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

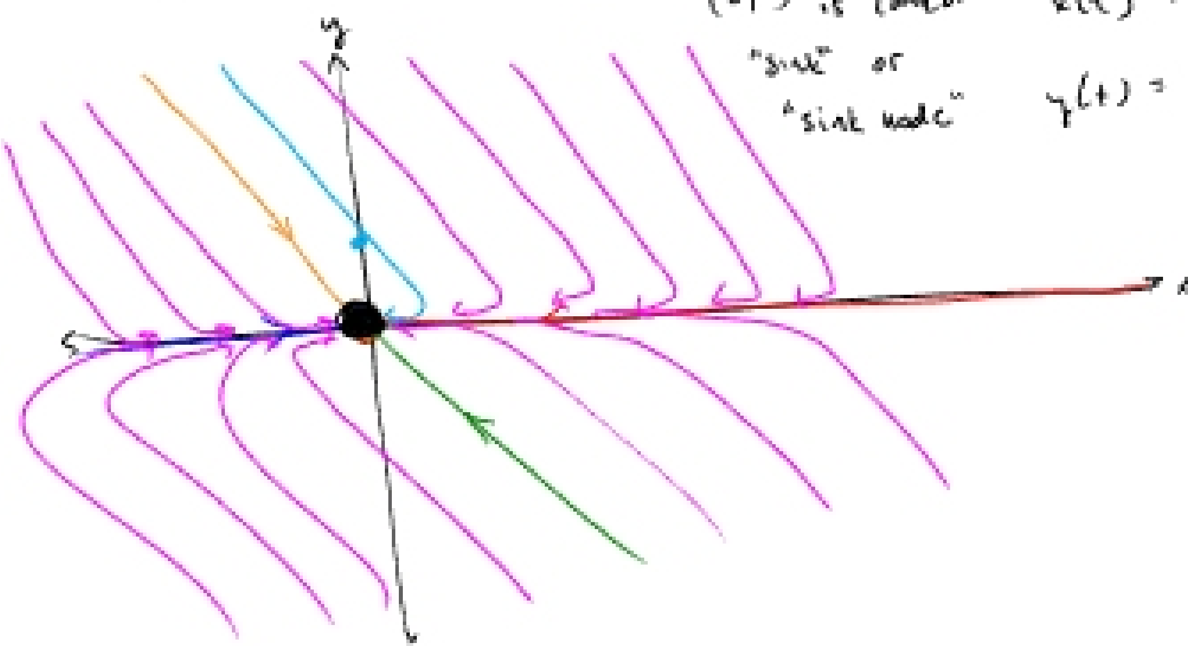
Ex: Sketch the solutions to the linear system

$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, sol: $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{-t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(0,0) is called "sink" or "sink node"

$x(t) = c_1 e^{-t} + c_2 e^{-3t}$

$y(t) = -c_2 e^{-3t}$



How to sketch?

- λ_1, λ_2 are real & distinct
- Plot the 4 "easy" solutions:

	c_1	c_2	$x(t), y(t)$
–	1	0	$e^{-t}, 0$
–	-1	0	$-e^{-t}, 0$
–	0	1	$e^{-3t}, -e^{-3t}$
			$3t \quad -3t$

0	1	e^{-t}, $-e$
0	-1	$-e^{-3t}$, e^{-3t}

$-\infty < t < \infty$

- $C_1 = 1$, $C_2 = -1$

$$\begin{bmatrix} x(t) = e^{-t} - e^{-3t} \\ y(t) = e^{-3t} \end{bmatrix}$$

$$\lim_{t \to \infty} \left[e^{-t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right]$$

dominant term

approaches 0 faster, so is negligible

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \approx e^{-t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lim_{t \to -\infty} \left[e^{-t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right]$$

negligible

dominant term

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \approx -e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \approx \begin{matrix} x = -e^{-3t} \\ y = e^{-3t} \end{matrix}$$

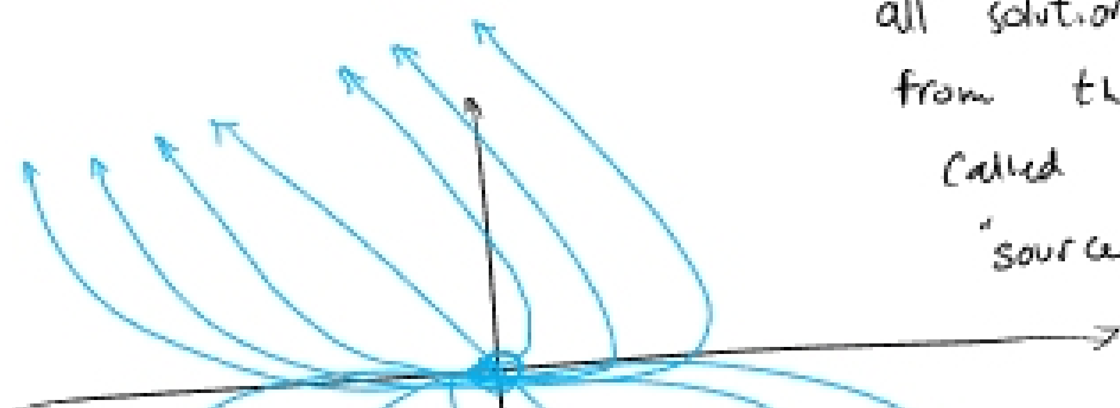
$\begin{bmatrix} \\ y(t) \end{bmatrix} \sim$

Ex 2: Multiply the matrix by -1

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\lambda = 1$
$\lambda = 3$

c_1	c_2	$x(t)$, $y(t)$
1	0	e^t, 0
-1	0	$-e^t$, 0
0	1	e^{3t}, $-e^{3t}$
0	-1	$-e^{3t}$, e^{3t}



all solutions emanate from the origin, called "source" or "source node"