Chapter5. Time Value of Money

* The Money Rules
#1: The more, the better => $2 today better than $1 today
#2: The sooner, the better => $1 today better than $1 tomorrow
#3: Tradeoff between the amount and the time
=> $1 today is equivalent to say, $1.06 tomorrow or $.96 today may be equivalent to $1 tom.
#4: Choose the point in time first, then try to come up with the equivalent $ amount
=> $100 today vs $109 tomorrow?
Today (Present Value): $100 today vs say $104.64 today's value (PV) of $109 tom.
OR Tomorrow (FV): say, $106 FV of $100 today vs. $109 tomorrow.
=> NEED PV->FV or FV->PV conversions just like any conversions to compare, add, subtract.
e.g, 25 inches vs 2 feet, note an inverse relationship (note the inverse relationship between the two conversion factors, x12 or x(1/12))

* TVM Fundamental Relationships
Note: (1) Interest rate per period vs. **APR (nominal rate, rate quoted)**
e.g., 6% for six months vs. 10% for a year? Which one is more? That is why we want to have the interest rates quoted on the same time length, a year. However, what is the actual interest earned? For all of our TVM calculations, we use interest rate per period (=periodic rate). How can I get the interest rate per period? Answer="APR/# of periods a year"
Ex: Quarterly Compounding, 10% APR, 10%/4=2.5% per period
(2) Simple interest vs. compound interest (interest on interest) in case of multiple prds?
Ex: Semi-annual example, 2 periods (two six months) with 12% APR?
What is the interest rate per period=12%/2=6%, What is your total interest for a year based on simple interest calculation method? 6% x 2 =12%.
Based on compounding? $(1+0.12/2)^2 = (1.06)^2=1.1236$ => 12.36% actual rate of return from compounding method. **Yield (APY) =actual rate of return vs. APR (Rate)**
(3) $FV = PV*(1+I)^N$ => Given any 3 variables out of 4 variables (FV, PV, I, and N), you can solve the equation for the 4th
Sometimes, we use i or r instead of I, and n instead of N, $(1+I)^N$ =FVIF(I%, N)
(4) **Timeline! I=interest per period!, N=#periods!** (p.124 – p.125)

1) Finding FV: $100 deposited for 3 years at 10%, semi-annual compounding? (p.125 – p.130)
(1) Using the formula (timeline!)
(2) Using the TVM(FV) table, FVIF(I%, N)=Conversion factor of[1$ today->FV] : note!
(3) Financial calculator: Check p/Y=1. Clear TVM each time before calculation.

Aside: **Rule of 72! How long it takes to double your money,**
#of periods to double my money=72/interest rate per period, Alternatively, you can use this to determine the interest rate required for you to double the money
Ex. 6% per year => 72/6=12 years, 10 years? Approx 7.2%. 7.18% Exactly
Ex. How long does it take to triple your money?

2) Finding PV (discounting): How much to deposit to have $100 3 years later at 10%, semi-annual compounding? (p.131 – p.133)
(1) Using the formula, Note that the inverse relationship, $PV=FV*\{1/(1+I)^N\}$
(2) Using the TVM(PV) table, $PVIF(I\%, N) = 1/(1+I)^N$
(3) Financial calculator:

3) Finding a time period: $100 deposited today and wish to have $370 at 16% with semi-annual compounding, # years? => get # of periods, then 8.5 years (p.134)

(1) Using the formula => 370=100*$(1+.08)^N$, # of years=N/2=17/2=8.5

(2) Using the TVM(maybe FV) table

(3) FC: Note "-" . Press CPT, then N

4) Finding an interest rate: $100 today and wish to have $1,700 in 12.5 years with semi-annual compounding, interest rate (APR)? => APR=12%*2=24% (p.133 – p.134)

(1) Using the formula and Trial & Error, 100=1700*$\{1/(1+I)^{25}\}$ or 1700=100*$(1+I)^{25}$

(2) Using the TVM (maybe FV) table

(3) FC: Note "-" . Press CPT, then and i/Y

* Multiple Cash Flows

1) Uneven Flows

=> Brutal Force (p.143 – p.146): Determine the PV(or FV) of each payment. Then add all the PVs to determine the PV of all the payments.

FC: Enter CF_0=0, CF_1=100, CF_2=200, CF_3=300, CF_4=400, CF_5=600 together with I/Y=5%, then determine NPV

Excel is an excellent choice in this case.

Enter each cell the payment, 100, 200, 300, 400, 600 then enter =npv(5%, address of 100: address of 600), hit "enter" to get 1,335

We can also Excel to back-figure the interest rate to produce the PV given. In other words, determine I/Y given PV and cash flows. In this example, have -1,335 on top of the cash flows (right above 100), then "=IRR(address of -1,335: address of 600)" and hit "enter" to get 5%.

2) Annuity: equal amount for some consecutive periods (p.134 – p.141)

(1) Different types – Ordinary (p.135)=each payment is made at the end of each period, Annuity Due (p.135)=each payment is made at the beginning of each period, Perpetuity(p.141 – p.142)=ordinary annuity, but forever

(2) Annuity examples (annuity, mortgage payments, lease payments, etc. vs. apartment rent)

(2) Analogous example: 3 yards(1760), 3 feet(5280), 3 inches(63360) on the line

(3) How to compute FV, PV of an ordinary annuity? (p.135, p.138)

Ex1: Save $100 each month for 10 years, at 6%, FV at the end of the 10th year? $16,388

How much extra money you have from this = 16,388 – 12,000 =4,388 due to interest.

Ex2; I would like to receive $100 each month for the next 10 years at 6%. How much do I have to save today? $9,007.35. Note that PV of $9,007.35, the annuity, and FV_{120} of $16,388 are all equivalent at 6% APR (or 0.5% per period).

a) Brutal Force = even for annuity, you can deal with each payment, then sum them up to get the total PV or FV. This is inefficient.

b) FVIFA(I%, N), PVIFA(I%,N): In our example above, I=0.5%, N=120

c) FC: note "-" PMT, N, I

(4) How to compute FV, PV of an annuity due? (p. 137: FV of an annuity due)

a) Brutal Force

b) FVIFA*(1+I), PVIFA*(1+I) => since you have each period one period earlier

c) FC: Change the setting to BEG => maybe, don't do it

d) Ex2: everything is the same as Ex1, except you pay (or get) $100 at the beginning of each month? PV=100*PVIFA(.5%, 120)*(1.05)=9,052.38. Alternatively, you can look at this as an ordinary annuity with 119 periods and $100 extra today =100*(.5%, 119)+100=9,052.38. How about FV?

FV=100*FVIFA(.5%, 120)*(1.05). Alternatively, you can determine the FV based on the PV you have already determined using the relationship between PV and FV, $FV=PV*(1.05)^{120}$.

(5) How to compute FV, PV of perpetuity? (p.141 – p.142: PV of perpetuity)

PV of perpetuity = C/I, C=payment per period, I=interest rate per period

Ex: PV of $100 monthly perpetuity at 12% APR= 100/0.01=10,000

3) Constant growth rate model

(1) What is it? $C_n=C_{n-1}*(1+g)$, Timeline

(2) PV equation and the formula $=C_1/(I-g)$

(3) Perpetuity is a special case with g=0% = C/I as we saw earlier

EX: C_0=$2, g=5%. Then $C_1=C_0*1.05$=$2.10, $C_2=C_1*(1+g)=C_0*(1+g)^2$, - -

$C_n=C_0*(1+g)^n=2*(1.05)^n$ - - > exponential growth.

PV of the constant growth rate payments = C1/(I-g)

EX: If I=10%? PV= $2.10/(0.10 – 0.05) =$42. What if everything is the same, but g=0% (perpetuity)?, then C1=Co*(1+0.0)=2, PV=2/0.1=$20. The extra $22 is for the growth of the future payments.

* APR / EAR (p.148 – p.150), effective annual rate = (annual percentage) yield (APY)

APR is a rate quoted, EAR is taking into the compounding effect

$EAR=(1 + APR/n)^n - 1$, n=# of periods a year

EX: APR=10%, Semiannual,

Semiannual => n=2 => $EAR=APY=(1.05)^2 – 1$ = 0.1025 or 10.25%

Quarterly => n=4 =>$EAR=(1.025)^4-1$ = 0.10381 or 10.381%

Frequent compounding produces a higher APY? Because of more interest on interest

* Loan Amortization (p.151 – p.152)

EX: House price=$300,000, 20% down, 30 year fixed at 6%, $10,000 closing cost (for paperwork, the point (? = extra payment upfront to buy a lower interest rate), etc)

How much to borrow? 300,000*(1-0.2)=240,000 + 10,000 = 250,000

Monthly payment=$1,498.88 (Note that you have an equal monthly amount, the composition between interest portion and the repayment of the principal varies over time. Initially, more for interest payments Table 5-4 on p.151).

Total payment=1,498.88*360=$539,597

Total interest payment=539,597 – 250,000 =289,597

What if I choose to have a 15 year loan instead?

Monthly payment=2,109.64, Total payment=2109.64*180 =379,735

Total interest payment = 379,735-250,000=129,735

⇨ Much lower interest payment for a 15 year loan. Is the 15 year loan better? Well, you need to look at your cash flow situation. Actually, the PVs of both are the same!

For TVM, it is important to have a time line first. Unfortunately, Aplia does not provide a time line presentation, makes it unnecessarily difficult to us to understand what is going on.

Please also remember, interest rate per period and # periods. "-" for cash outflows. However, need to convert your answers to APR and # years.