General Physics - E&M (PHY 1308) Lecture Notes

Lecture 016: The Magnetic Field due to Moving Charge

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no tags

What is Magnetism?!

It was Hans Christian Oersted who is credited with setting physics and chemistry on the path to an understanding of magnetism. While magnetism has been observed since about 500-600 BC (in the Western world, it was Aristotle who gave credit for the discovery of this phenomenon to a man named Thales), it was not at all understood until the 1800s. Oersted accidentally observed that an electric current caused a compass needle to deflect. We can easily reproduce this experiment. Magnetism, therefore, seems to have something to do with the MOTION of electric charge. Not long after Oersted's publicized observation, two French scientists - Baptiste Biot and Felix Savart - performed experiments and determined the exact form of the force law for a steady current. We call this the *Biot-Savart* Law, and we'll explore it now.

The *Biot-Savart* law considers a steady current moving through a conductor. There are similarities and differences between *B-S* and Coulomb's Law. Let's write Coulomb's Law for a small piece of a distribution of charge:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

Draw a picture representing the situation that can be described by Coulomb's Law (a blob of charge, considering the electric field due to a piece of the blob).

Now draw a picture representing the situation we want to analyze in magnetic fields. We want to know the field, $\vec{B}$, at a point P some distance, r, from a part of the conductor ($d\vec{L}$) carrying a steady current I. Our convention will again be that $\hat{r}$ points from the conductor element to the point P.

Now let's think about the differences between the static charge situation considered by Coulomb's Law and the moving charge situation in *B-S* Law:

- In the BSL, we are considering a current element, $Id\vec{L}$, which is a vector quantity (it flows along a direction in the conductor). In CL, we were considering a piece of charge which had no direction. It was just a number.
- In the BSL, the source of magnetic field is a VECTOR quantity - current times $d\vec{L}$. In the CL, the source of electric field is a scalar quantity - charge. We have to account for direction of motion in the BSL.
- In the BSL, the field contribution of $Id\vec{L}$ depends on the orientiation of the conductor to the unit vector - it depends on the sine of the angle, specifically. In CL, we had no such oddity.

The *B-S* Law, which describes all of our observations, is as follows:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L}\times\hat{r}}{r^2}$$

Here, we have a new constant that has been determined from experimental measurement: μ_0, the **permeability constant**. It's EXACT value is $4\pi \times 10^7 \mathrm{N}/\mathrm{A}^2$. Equivalent units are often used: $\mathrm{T}\cdot\mathrm{m}/\mathrm{A}$.

There is one other important distinction between the BSL and CL. The CL gives us the electric field in terms of isolated charge elements. But it's impossible to talk about an isolated current element, because it necessarily must be part of a circuit. In order to get the total magnetic field, you have to integrate around the entire circuit to get the magnetic field at point P. Because magnetic field is a vector, it obeys the superposition principle so we just have to add up all the current elements:

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{L}\times\hat{r}}{r^2}$$

The above is the *integral form* of the BSL. The magnetic field thus depends on the details of the current distribution. Generally speaking, though, the cross-product in this law tells us that magnetic field lines encircle the path of the current perpendicular to its direction. Here we have another version of the right-hand rule:

- To determine the direction of magnetic fields around a conductor, point your thumb in the direction of current. The direction your fingers would curl around the conductor indicates the direction of magnetic field lines.

Example: magnetic field around a straight conducting wire

Consider an infinitely long straight wire carrying a steady current I. Find the magnetic field at a point P which lies a distance y above the wire.

- Draw the wired and setup the problem. Use the new right-hand rule to note which direction (into or out of the board) we expect the field to point.

Begin by writing the BSL:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{L} \times \hat{r}}{r^2}$$

- Let's choose CARTESIAN coordinates since we have all these handy straight lines in the problem and because whatever the distance y of point P above the line, it's fixed no matter where along the conductor we are considering.

What are the unknowns?

- We need to sort out expressions for $\hat{r}$, $d\vec{L}$, and the cross-product in terms of the geometry (coordinates) of the problem
- We need an expression for r^2 in terms of geometry.

Let's attack pieces. What are they? They are: (1) the distance, r, (2) the relationship between dL and the geometry of the coordinate system, (3) the direction of the magnetic field due to $d\hat{L} \times \hat{r}$, and (4) the magnitude of $Id\vec{L} \times \hat{r}$.