# Period #10: Multi−dimensional Fluid Flow in Soils (II)

A. Review

- Our objective is to solve multi−dimensional fluid flow problems in soils.
- Last time, mass conservation and Darcy's Law were used to derive the so−called *Laplace Equation* which governs seepage in homogeneous, isotropic soil deposits.

$$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 + \partial^2 h/\partial z^2 = 0$$

B. Possible Methods for Solving the Laplace Equation.

1) Analytical, closed form or series solutions of the PDE.
   - quite mathematical, and not very general.

2) Numerical solution methods
   - typically, the *finite element method* or the *finite difference method.*
   - very powerful and easy to apply
   - can deal with heterogeneity, anisotropy, 2D, 3D
   - Will use *finite element method* in Lab 6.

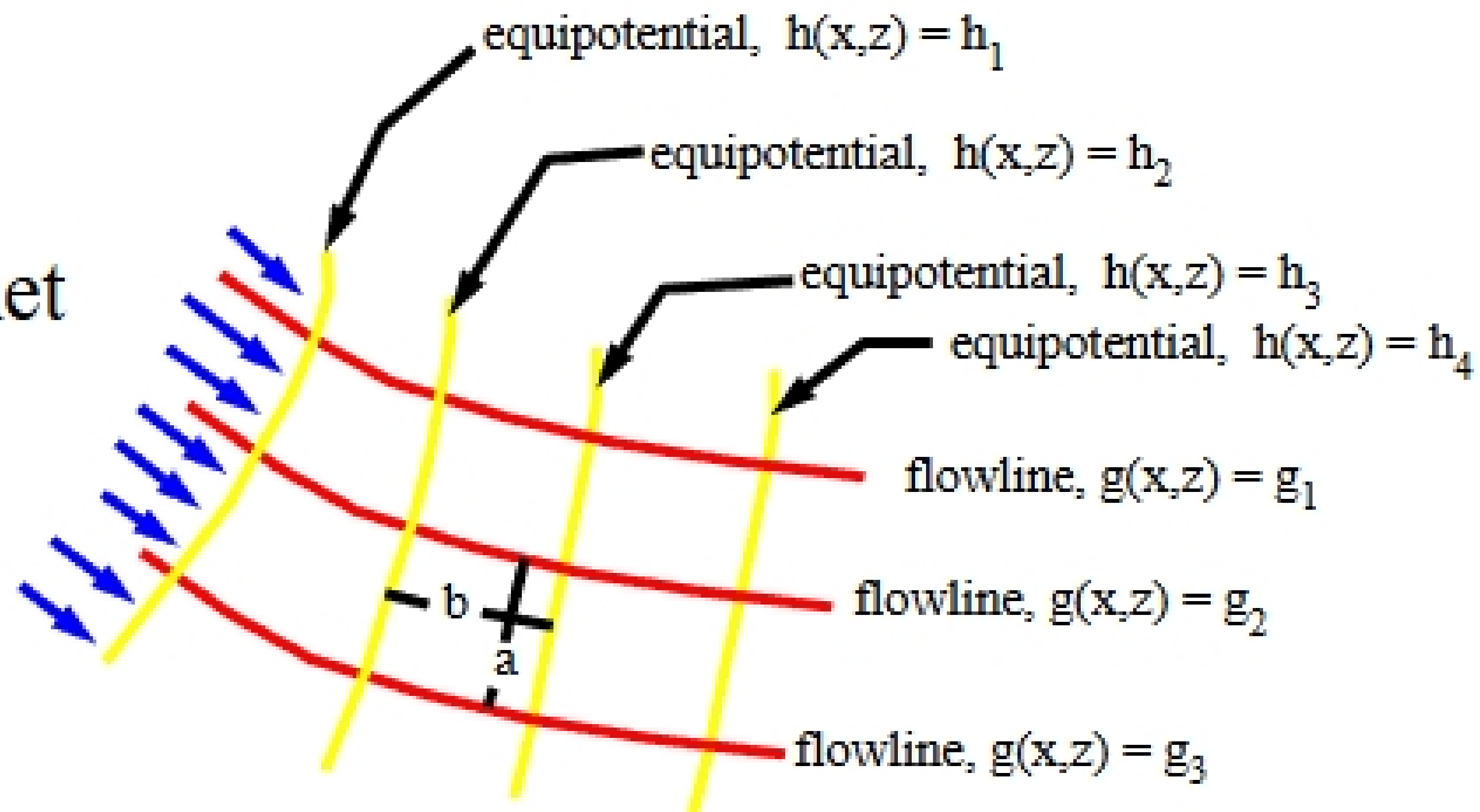
3) Graphical Techniques − *Flow−net Methods*
   - commonly used in engineering practice to solve 2D flow problems.
   - the ideas behind this method are now explained.

## C. Flow−net Methods

- straightforward graphical method to solve 2D seepage problems.
- underlying idea:
  - solutions of Laplace Equation consist of two families of orthogonal curves in the (x,z) plane. These families of curves make a flow net.

  - equipotentials: $h(x,z) = c$: family of curves along which head is constant
  - flow lines : $g(x,z) = d$: family of curves across which flow does not occur
  - $h$ and $g$ curves must intersect at right angles wherever they cross.
  - two $h$ curves cannot intersect each other; two $g$ curves cannot intersect.

Partial Flow−net



- Consider the flow rate $\Delta q$ though a given rectangle formed by two h–curves and two g–curves.
  - since seepage is occuring parallel to the g–curves, can use 1–D form of Darcy's Law

$$\begin{aligned}\Delta q &= k\, i\, a \\ &= k\,(\Delta h/b)\, a \\ &= k\,\Delta h\,(a/b)\end{aligned}$$

- In practice, a net of equipotentials (h–curves) and flow lines (g–curves) are drawn on the flow domain such that:

  a) The soil domain is drawn to scale;
  b) The boundary conditions are clearly identified (for example, are the boundaries of the flow domain equipotentials or flow lines?);
  c) The cells formed by intersecting families of curves are all approximately square with ratios $(a/b) \sim 1$.
  d) The equipotentials and flow–lines are orthogonal, wherever they intersect.

- Drawing good flow nets that satisfy these criteria is not always easy. Usually it takes a fair amount of trial and error ( and a pencil with a good eraser !).

- If flow nets can be drawn satisfying these requirements, then:
  1) the flow in each channel will contain an equal flow. (A channel is the region between two flow lines or g– curves.)
  2) the head drop between all adjacent equipotentials or h–curves is the same.