11/6/14

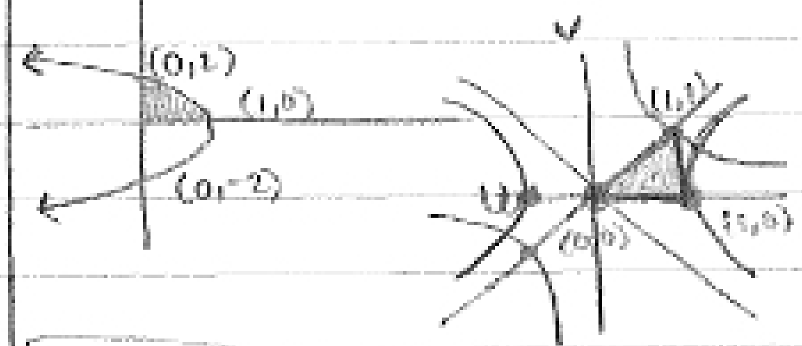
Example Prom 15.8

$x = u^2 - v^2$

$y = 2uv$

$$\int_0^1\int_0^{2\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dy\, dx = \iint \sqrt{(u^2-v^2)^2+(2uv)^2} \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 2u & -2v \\ 2v & 2u \end{pmatrix} = 4u^2 + 4v^2$$



$y^2 = 4 - 4x \quad 4x = 4 - y^2$

$x = 1 - \frac{y^2}{4}$

point $\begin{cases} 0 = u^2 - v^2 \\ 2 = 2uv \end{cases}$ $\quad uv = 1$

$\boxed{u = v}$

$x = u^2 - v^2 = 0$

$y = 2u^2$

$\left.\begin{matrix} u^2 - v^2 = 1 \\ 2uv = 0 \end{matrix}\right\}$ point

$\boxed{v = 0}$ $x = u^2 \quad y = 0$

$\boxed{u = 1}$

$x = 1 - v^2$

$y = 2v$

$4x = 4 - y^2$

$4(1-v^2) = 4 - (2v)^2$

$4 - 4v^2 = 4 - 4v^2$ ✓

$$\int\int \sqrt{(u^2-v^2)^2+(2uv)^2} \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv$$

$$\int_0^1\int_v^1 \sqrt{(u^2+v^2)^2}\ (4u^2+4v^2)\, du\, dv$$

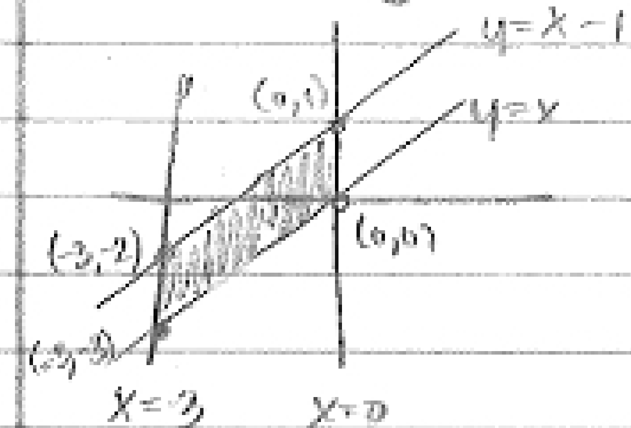
$$\int_0^1 \int_v^1 (u^2+v^2)(4u^2+4v^2)\, du\, dv$$

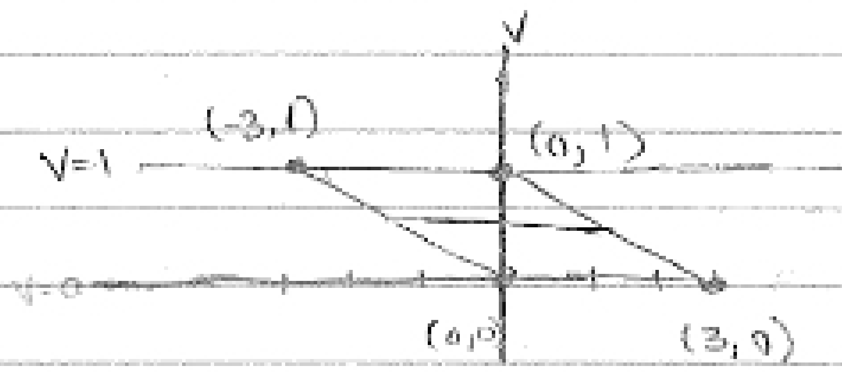
$$\boxed{\int_0^1\int_v^1 4(u^2+v^2)^2\, du\, dv}$$

$X = -3 \quad x = 0 \qquad y = x \qquad y = x + 1 \quad \leftarrow R$

$u = 2x - 3y$

$v = -x + y \qquad \iint_R 2(x-y)\, dx\, dy$





(x y)	u v
(0,1)	(-3,1)
(0,0)	(0,0)
(-3,-2)	(0,1)
(-3,-3)	(3,0)

$u + 3v = 3$

$u + 3v = 0$

$$\int_0^1 \int_{-3v}^{3-3v} 2(-v)$$

$\frac{\partial(x\,y)}{\partial(u\,v)} = 1$

$$\frac{\partial uv}{\partial(xy)} \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix} = |-1| = + \quad \boxed{\int_0^1 \int_{-3v}^{3-3v} -2v \, du\, dv}$$