Good Practice Problem!

$$e^{x^2+y+z^2} + \cos(x,y) = 5$$

Dependent: x
Independent: y, z

$$\frac{\partial x}{\partial y} = e^{x^2+y+z^2}\left(\frac{\partial}{\partial y}(x^2+y+z^2)\right) - \sin(xy)\frac{\partial}{\partial y}(xy) = 0$$

$$= e^{x^2+y+z^2}\left(2x\frac{\partial x}{\partial y}+1\right) - \sin(xy)\left(\frac{\partial x}{\partial y}y + x\right) = 0$$

$$= \left[e^{x^2+y+z^2}2x - \sin(xy)y\right]\frac{\partial x}{\partial y} + \left[e^{x^2+y+z^2} - \sin(xy)x\right] = 0$$

$$e^{x^2+y+z^2}2x - \sin(xy)y]\frac{\partial x}{\partial y} = -e^{x^2+y+z^2} - \sin(xy)x$$

$$\frac{\partial x}{\partial y} = \frac{-\left[e^{x^2+y+z^2} - x\sin(xy)\right]}{\left[e^{x^2+y+z^2}2x - y\sin(xy)\right]}$$

9/22/14

linear approximation
-tangent spaces
directional derivatives.

Remark $f: \mathbb{R}^n \to \mathbb{R}^m$ and $P \in \mathbb{R}^n$
- then the derivative $f'(P)$ defines a linear function

$$L: \mathbb{R}^n \to \mathbb{R}^m$$
$$Q \mapsto f'(P)\,Q$$

L = linear - means:
1.) $L(Q+R) = L(Q) + L(R)$
2.) $L(Q\alpha) = L(Q)\alpha$
α = a number

EX) $(f: \mathbb{R}^1 \to \mathbb{R}^1)$ $f(x) = x^3$ $p = 2,$ $f'(x) = 3x^2$ $f'(2) = 12$

$L: \mathbb{R}^1 \to \mathbb{R}^1$

$Q \mapsto 12Q$

$Q = 5$, $R = 8$

$L(Q+R) = 12(5+8) = 156$
$L(Q) + L(R) = 12(5) + 12(8) = 156$ > they equal! yay!

$\alpha = 4$

$L(Q\alpha) = 12(5 \cdot 4) = 240$
$L(Q)\alpha = 12(5)(4) = 240$ > they equal! Yay!

$g: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

$g(u,v) = \begin{bmatrix} u^2 v \\ uv \\ u-v \end{bmatrix}$ $p = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $Q = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $R = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$ $\alpha = 4$

$g'(u,v) = \begin{bmatrix} 2uv & u^2 \\ v & u \\ 1 & -1 \end{bmatrix}$ $g'(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

$L(Q+R) = g'(1,2)\left[\binom{5}{2} + \binom{8}{7}\right]$

$= \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 13 \\ 9 \end{bmatrix} = \begin{bmatrix} 52+9 \\ 26+9 \\ 13-9 \end{bmatrix} = \begin{bmatrix} 61 \\ 35 \\ 4 \end{bmatrix}$

$L(Q) + L(R) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 61 \\ 35 \\ 4 \end{bmatrix}$

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linear approximations

Given $f: \mathbb{R}^n \to \mathbb{R}^m$ & $P \in \mathbb{R}^n$ & $\Delta P \in \mathbb{R}^n$

$f(P+\Delta P) \approx f(P) + f'(P)\Delta P$ + error term.

know ... $\frac{|\varepsilon_p(P,\Delta P)|}{|\Delta P|} \to 0$ as $\Delta P \to 0$

$f: \mathbb{R}^1 \to \mathbb{R}^1$ $\quad P = 2 \quad f'(2) = 12 \quad$ set $\Delta P = .03$

$x \mapsto x^3$

$f(2.03) \approx f(2) + f'(2)(0.03)$

$8 + 12(.03)$

$f(2.03) = 8.36 \quad \to$ Actual $= 8.365427$

$g: \mathbb{R}^2 \to \mathbb{R}^3 \qquad (u,v) \mapsto \begin{bmatrix} u^2 v \\ uv \\ u-v \end{bmatrix} \qquad P = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$g(1,2) = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \qquad g'(1,2) = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

$\Delta P = \begin{bmatrix} .02 \\ -.03 \end{bmatrix}$

$$g(1.02, 1.97) \approx \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} .02 \\ -.03 \end{bmatrix}$$

$$\approx \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} .05 \\ .01 \\ .05 \end{bmatrix} \quad \begin{bmatrix} 2.05 \\ 2.01 \\ -.95 \end{bmatrix}$$