9/11/14 MATH 2850

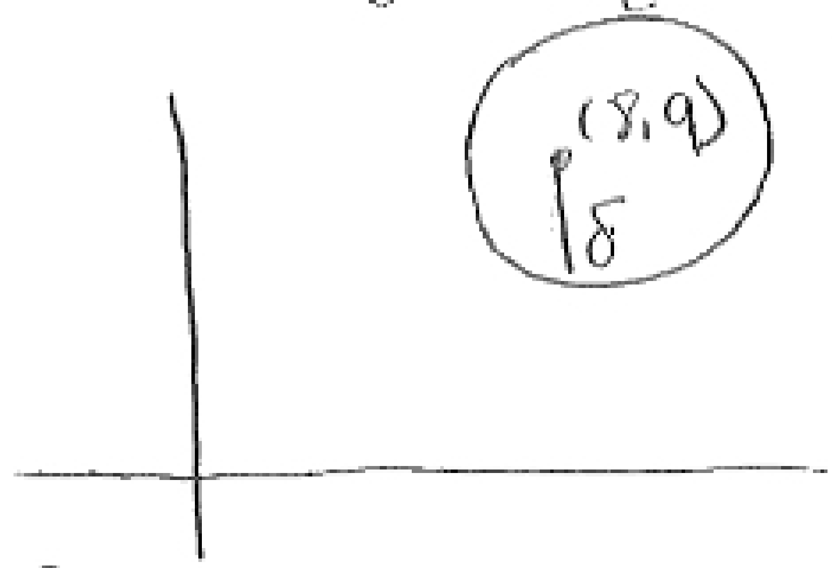
Ch. 14

- limits and continuity

$\lim_{x \to 3} f(x) = 7$ means when is x is "close" to 3 then f(x) is "close" to 7

Given $\varepsilon > 0$ we have to produce $\delta > 0$ so that:

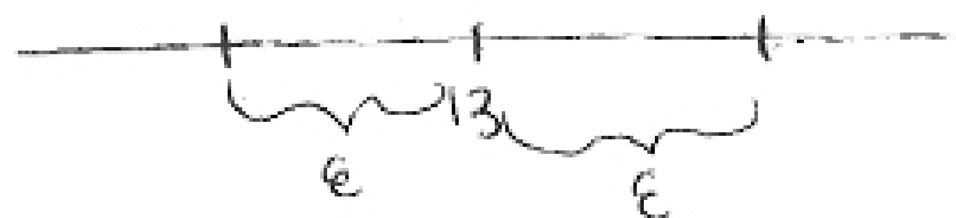
$\lim_{(x,y) \to (8,9)} f(x,y) = 13$



*being "close" to (8,9) is being inside disk.

Continuous at $x = x_0$ if:

1) has to exist, value at point
2) limit at point
3) value and limit have to be equal



$f(x) = \frac{x^3 + x^2 + 2x}{x^4 + x^4 + 3x^2}$ except at $x = 0$

$g(x) = \frac{\sqrt{x^2 - 4}}{x^2 + x^4}$ $\quad x^2 - 4 \geq 0$

$x \geq 2 \quad x \leq -2$

$f(x) = \frac{x^3 + x + 5}{x^4 + x^2}$ $\quad \lim_{x \to 1} f(x) = f(1) = \frac{7}{2}$ ✓

- How to find the limit of a function if the function is not defined at the point. $\lim_{P \to P_0} F(P)$ but $f(P_0)$ DNE

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y-xy+4x^2-4x}$$

$$\frac{0}{4(-4)-(2)(-4)+4(2)^2-4(2)} = \frac{0}{-16+8+16-8} = \frac{0}{0}$$

① Do some algebra $f(P) = \frac{g(P)}{g(P)} h(P)$ where $h(P)$ is defined.

$$\lim_{x\to 0} \frac{x^2}{x} = \lim_{x\to 0} \frac{x}{x} x = \lim_{x\to 0} x = 0$$

② Do a change of variables (maybe polar) - maybe that's easier

③ Do ε-δ argument (not very common)

$$\frac{y+4}{x^2y-xy+4x^2-4x} = \frac{\text{something}}{\text{something}} (\text{else}) = \frac{y+4}{y+4}\left(\frac{1}{x^2-x}\right)$$

$$\begin{array}{r|l} & x^2-x \\ \hline y+4 & x^2y-xy+4x^2-4x \\ & yx^2-yx+4x^2-4x \\ \hline & 0 \end{array}$$

$$\lim_{(x,y)\to(2,-4)} \frac{\cancel{y+4}}{\cancel{y+4}}\left(\frac{1}{x^2-x}\right)$$

$$\lim_{(x,y)\to(2,-4)} \frac{1}{x^2-x} = \frac{1}{4-2} = \boxed{\frac{1}{2}}$$

ex: $\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-(y+1)} = \frac{0}{0}$ $\quad x-(y+1) = \sqrt{x}^2 - \sqrt{y+1}^2 = (\sqrt{x}-\sqrt{y+1})(\sqrt{x}+\sqrt{y+1})$

$$\lim_{(x,y)\to(4,3)} \frac{\cancel{\sqrt{x}-\sqrt{y+1}}}{(\cancel{\sqrt{x}-\sqrt{y+1}})(\sqrt{x}+\sqrt{y+1})} = \frac{1}{\sqrt{x}+\sqrt{y+1}} = \boxed{\frac{1}{4}}$$

$$\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2}$$

*when going to (0,0) <u>sometimes</u> switching to polar is helpful

$$x = r\cos\theta \quad y = r\sin\theta$$

$$\lim_{(r)\to 0} \frac{2r\cos\theta}{r^2\cos^2\theta + r\cos\theta + r^2\sin^2\theta} \Rightarrow \frac{2r\cos\theta}{r^2 + r\cos\theta}$$

$$\lim_{r\to 0} \frac{\cancel{r}}{\cancel{r}} \cdot \frac{2\cos\theta}{r+\cos\theta} = 2$$