Prof. Charlie Odenthal
MATH 2850

8/26/14

vector value functions

$r: \mathbb{R} \longrightarrow \mathbb{R}^3$

$t \longmapsto \langle x, y, z \rangle$ ← vector

ex:

$r: \mathbb{R} \longrightarrow \mathbb{R}^3$

$t \longmapsto \langle t^3, t^2+4, 3t^2-1 \rangle$

$x(t) = t^3$

$y(t) = t^2+4$

$z(t) = 3t^2-1$

$x\hat{i} + y\hat{j} + z\hat{k}$ ⟵ only use this when taking * the cross product

$$r'(t) = \frac{dr}{dt} = \left\langle \frac{d}{dt}(t^3), \frac{d}{dt}(t^2+4), \frac{d}{dt}(3t^2-1) \right\rangle$$

$$r'(t) = \langle 3t^2, 2t, 6t \rangle$$

$$r(t+h) = r(t) + r'(t)h + E(t+h)$$

where $\frac{|E(t+h)|}{h} \to 0$ as $|h| \to 0$

$r(t) = \langle t^3, t^2+4, 3t^2-1 \rangle$

$r(t+h) = \langle (t+h)^3, (t+h)^2+4, 3(t+h)^2-1 \rangle$

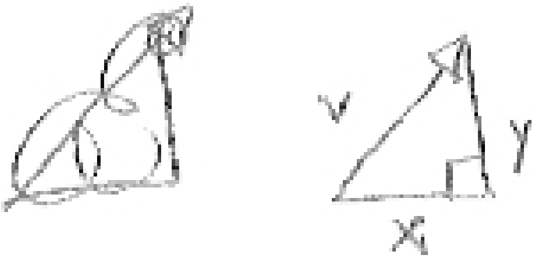
$= \langle t^3+3t^2h+3th^2+h^3, t^2+2th+h^2+4, 3(t^2+2th+h^2)-1 \rangle$

$E(t,h) = r(t+h) - r(t) - r'(t)h$

$= \langle \cancel{t^3}+\cancel{3t^2h}+3th^2+h^3-\cancel{t^3}-\cancel{3t^2h}, \cancel{t^2}+\cancel{2th}+h^2+\cancel{4}-(\cancel{t^2}+\cancel{4})-\cancel{2th},$
$3t^2+\cancel{6th}+3h^2-\cancel{1}-(\cancel{3t^2}-\cancel{1})-\cancel{6th} \rangle$

$= \langle 3th^2+h^3, h^2, 3h^2 \rangle = E(t,h)$

$= h^2\langle 3t+h, 1, 3 \rangle$



$|v| = \sqrt{x^2+y^2}$ ← finding magnitude.

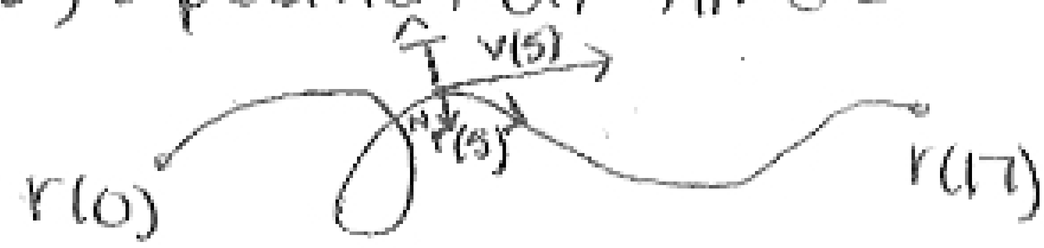
$|kv| = |k|\,|v|$

scalar → k; vector → v; $|k|$ ← absolute value; $|v|$ ← length

$$|E(t,h)| = h^2\sqrt{(3t+h)^2+(1)^2+(3)^2}$$

$$\frac{|E(t,h)|}{|h|} = |h|\underbrace{\sqrt{(3t+h)^2+10}}_{\text{goes to } 3t^2+10} \rightarrow 0 \text{ as } |h| \rightarrow 0$$

r(t) : position at time t



velocity : $v(t) = r'(t) = \langle 3t^2, 2t, 6t \rangle$

acceleration: $a(t) = v'(t) = r''(t) = \langle 6t, 2, 6 \rangle$

Speed = magnitude of velocity

$= |v|$

$= \sqrt{(3t^2)^2+(2t)^2+(6t)^2}$

$= \sqrt{9t^4+40t^2} = |t|\sqrt{9t^2+40}$

*abs. value signifies magnitude.

*velocity is tangent to the curve of position

$$T = \frac{v}{|v|} = \left\langle \frac{3t^2}{|t|\sqrt{9t^2+40}}, \frac{2t}{|t|\sqrt{9t^2+40}}, \frac{6t}{|t|\sqrt{9t^2+40}} \right\rangle$$

*don't try to simplify it

$$\frac{d}{dt}\left(28\, r(t)\right) = 28\, r'(t)$$

$s: \mathbb{R} \longrightarrow \mathbb{R}^3$
$t \longmapsto \langle t^5, t^4, t^3 \rangle$

$$\frac{d}{dt}(r+s)(t) = r'(t) + s'(t)$$
$$= \langle 3t^2, 2t, 6t \rangle + \langle 5t^4, 4t^3, 3t^2 \rangle$$
$$= \langle 3t^2 + 5t^4, 2t + 4t^3, 6t + 3t^2 \rangle$$

<u>Dot Product:</u>

$s: \mathbb{R} \longrightarrow \mathbb{R}^3$ $\qquad$ $r: \mathbb{R} \longrightarrow \mathbb{R}^3$
$t \longmapsto \langle u, v, w \rangle$ $\qquad$ $t \longmapsto \langle x, y, z \rangle$

$$\frac{d}{dt}(r \cdot s) = \frac{d}{dt} = \frac{d}{dt}(xu + yv + zw)$$
$$= \frac{d}{dt}(xu) + \frac{d}{dt}(yv) + \frac{d}{dt}(zw)$$
$$= (x'u + xu') + (y'v + yv') + (z'w + zw')$$
$$= (x'u + y'v + z'w) + (xu' + yv' + zw')$$
$$= \langle x', y', z' \rangle \cdot \langle u, v, w \rangle + \langle x, y, z \rangle \cdot \langle u', v', w' \rangle$$
$$= r' \cdot s + r \cdot s'$$

<u>Cross product:</u>

$$\frac{d}{dt}(r \times s) = r' \times s + r \times s'$$

*do not interchange order!

$$r \times s = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ u & v & w \end{vmatrix} = \hat{\imath}\begin{vmatrix} y & z \\ v & w \end{vmatrix} - \hat{\jmath}\begin{vmatrix} x & z \\ u & w \end{vmatrix} + \hat{k}\begin{vmatrix} x & y \\ u & v \end{vmatrix}$$

$$= \langle yw - vz, -(xw - uz), xv - uy \rangle$$