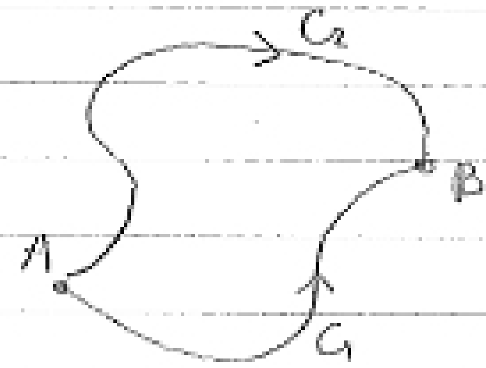


$$\int_{C_1} M\,dx + N\,dy = \int_{C_2} M\,dx + N\,dy$$

$$\int_{C_1} M\,dx + N\,dy - \int_{C_2} M\,dx + N\,dy$$

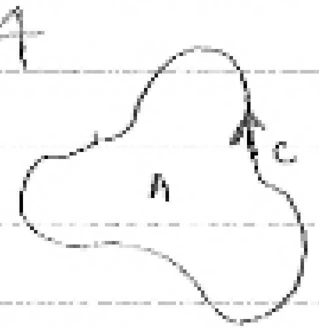
$$= \int_{C_1} M\,dx + N\,dy + \int_{-C_2} M\,dx + N\,dy$$

$$= \int_{C_1 - C_2} M\,dx + N\,dy = 0$$

11/20/14

Green's Theorem

$$\int_C M\,dx + N\,dy$$
$$= \iint_A N_x - M_y \; dA$$



Corollary: If $M_y = N_x$ then $\int_C M\,dx + N\,dy = 0$ for every simple closed curve C.

Last time: Proposition

If $\int_C M\,dx + N\,dy = 0$ for all simple closed curves then $\int_C M\,dx + N\,dy$ is independent of path.

EX) Consider $(x^2+y^2)\,dx + (x^3+y)\,dy$

Curve

$N_x = 3x^2$

$M_y = 2y$

$x = x^2 - 2$

$0 = x^2 - x - 2$

$(x-2)(x+1)$

$x = 2, -1$

$$\iint_A N_x - M_y \; dA = \boxed{\int_{-1}^{2}\int_{x^2-2}^{x} 3x^2 - 2y \quad dy\,dx}$$

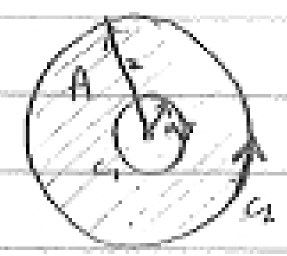
$$\int 3x^2y - y^2 \Big|_{x^2-2}^{x} dx$$

$$\int 3x^3 - x^2 - (3x^4 - 6x^2 - (x^4 - 4x^2 + 4)) \quad dx$$

$$\frac{3}{4}x^4 - \frac{x^3}{3} - \left(\frac{3}{5}x^5 - 2x^3 - \left(\frac{x^5}{5} - \frac{4}{3}x^3 + 4x\right)\right) \Big|_{-1}^{2}$$

= some #

EX) $(7x+y)dx + (11x+7y)dy$ (Not exact b/c $1 \neq 11$)



$$\int_{C_1+C_2} Mdx + Ndy = \int_{C_1} Mdx + Ndy + \int_{C_2} Mdx + Ndy$$

$$= -\int_{-C_1} Mdx + Ndy + \iint_{A_2} N_x - M_y \, dA$$

radius 1 = 1

radius 2 = 2

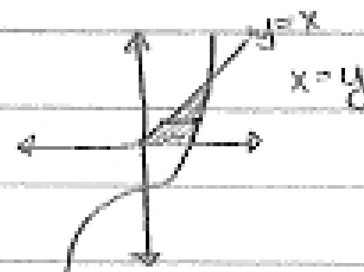
$$= -\iint_{A_1} N_x - M_y \, dA + \iint_{A_2} N_x - M_y \, dA$$

$$\int_{C_1+C_2} Mdx + Ndy = \iint_{A_2 - A_1} N_x - M_y \, dA$$

$$= \iint_A 11 - 1 \, dA$$

$$= 10 \iint_A dA = 10(\pi(2)^2 - \pi(1)^2) = \boxed{30\pi}$$

EX) $y = x \quad y = x^3 - 1 \quad y = 0$



$y = x^3 - 1 \quad x = (y+1)^{1/3}$

1-form: $(x^2 - x - 1)dx + (x+3)dy$

$$\int_C Mdx + Ndy = \iint_A N_x - M_y \, dA$$

$1 - 0$

$$= \int_0^{\alpha} \int_y^{(y+1)^{1/3}} 1 \, dx \, dy$$

$$= \int_0^{\alpha} (y+1)^{1/3} - y \, dy$$

$$= \tfrac{3}{4}(y+1)^{4/3} - \tfrac{1}{2}y^2 \Big|_0^{\alpha}$$

$$= \tfrac{3}{4}(\alpha+1)^{4/3} - \tfrac{1}{2}\alpha^2$$

$$\int_C Mdx + Ndy = \iint_A N_x - M_y \, dA$$