$$= 2\int_{-3}^{3} 2\sqrt{9-x^2}\,(2/3)(9-x^2)\;dx$$

$$= \frac{4}{3}\int_{-3}^{3}(9-x^2)^{3/2}\,dx \qquad \cancel{u = 9-x^2} \quad \cancel{du = -2x\,dx}$$

$$= \frac{4}{\cancel{3}}\int_{-\pi/2}^{\pi/2} 27\cos^3\theta\cdot\cancel{3}\cos\theta\,d\theta$$

$$(27)(4)\int_{-\pi/2}^{\pi/2}\cos^4\theta\,d\theta$$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$9-x^2 = 9-9\sin^2\theta = 9(1-\sin^2\theta)$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

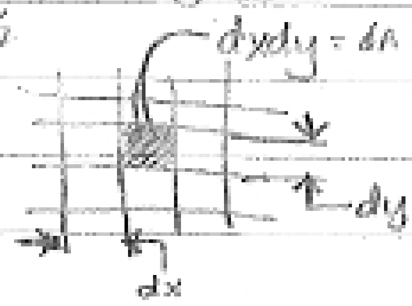
This is crazy! Switch to polar!

10/21/14

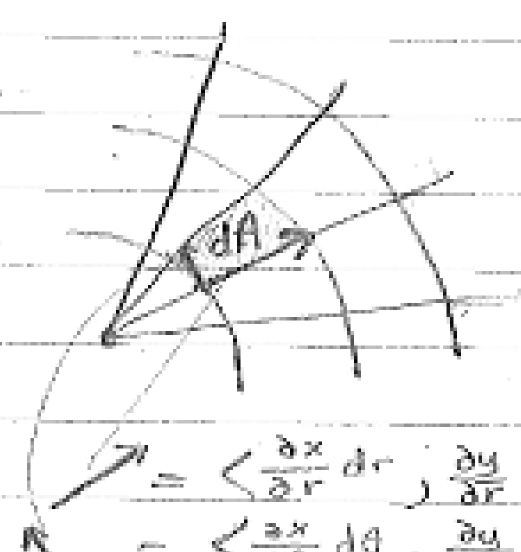
$x^2+y^2 \le 3$

Same problem as Monday! Just using polar coordinates instead.

$\int_{-\pi/2}^{\pi/2}\cos^4\theta\,d\theta$ ← thats ugly.



Polar:



$\to = \left\langle \frac{\partial x}{\partial r}dr, \frac{\partial y}{\partial r}dr \right\rangle$

$\nwarrow = \left\langle \frac{\partial x}{\partial \theta}d\theta, \frac{\partial y}{\partial \theta}d\theta \right\rangle$

$A = |\vec{v}||\vec{w}|\sin\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$\to = \langle \cos\theta\,dr, \sin\theta\,dr\rangle$

$\nwarrow = \langle -r\sin\theta, r\cos\theta\rangle$

$\langle \cos\theta\,dr, \sin\theta\,dr\rangle \times \langle -r\sin\theta, r\cos\theta\rangle$

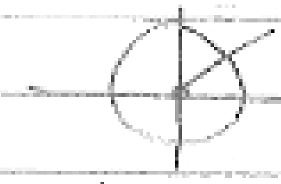
next page...

$$= \begin{vmatrix} i & j & k \\ \cos\theta\, dr & \sin\theta\, dr & 0 \\ -r\sin\theta\, d\theta & r\cos\theta\, d\theta & 0 \end{vmatrix} = \langle 0, 0, r\cos^2\theta\, dr\, d\theta + r\sin^2\theta\, dr\, d\theta \rangle$$

$$= \langle 0, 0, \underbrace{r\, dr\, d\theta}_{dA} \rangle$$

$dA = r\,dr\,d\theta$ r is the jacobian.

Back to $z = 9 - x^2 - y^2$



$x^2 + y^2 \le 9$

$$\iint_A 9 - x^2 - y^2 \, dA \Rightarrow \int_0^{2\pi}\int_0^{3} (9 - r^2)\, r\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{3} (9r - r^3)\, dr\, d\theta$$

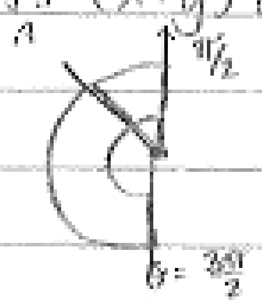
$$\int_0^{2\pi} \frac{9}{2}r^2 - \frac{1}{4}r^4 \Big|_{r=0}^{3} d\theta$$

$$\int \frac{81}{2} - \frac{81}{4} - 0 \, d\theta$$

$$\frac{81}{4}\int_0^{2\pi} d\theta$$

$$\left(\frac{81}{4}\right) \theta \Big|_0^{2\pi} = \frac{81\pi}{2}$$

$\iint_A (x+y)\, dA$ where A is to the left of y-axis and between circles $x^2+y^2=1$ and $x^2+y^2=4$



$r = 1$ & $r = 2$

$$\int_{\pi/2}^{3\pi/2}\int_1^2 (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

$$\iint (\cos\theta + \sin\theta)\, r^2\, dr\, d\theta$$

$$\int_{\pi/2}^{3\pi/2} (\cos\theta + \sin\theta)\, \frac{r^3}{3}\Big|_1^2 d\theta \Rightarrow \int_{\pi/2}^{3\pi/2} (\cos\theta + \sin\theta)\left(\frac{7}{3}\right) d\theta$$

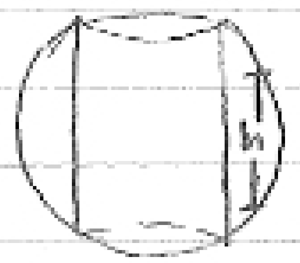
$$= \sin\theta - \cos\theta \Big|_{\pi/2}^{3\pi/2} \left(\frac{7}{3}\right)$$

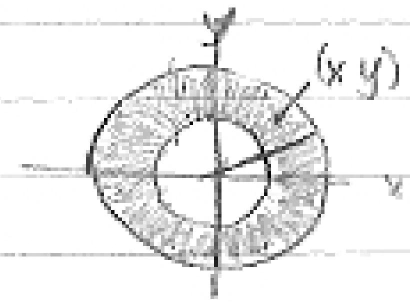
$$(-1) - (1)$$

$$(-2)\left(\frac{7}{3}\right) = \boxed{-14/3}$$

NOW SOMETHING MORE COMPLEX

Volume inside sphere $x^2+y^2+z^2=16$
outside cylinder $x^2+y^2=4$





$$\iint_A 2\sqrt{16-x^2-y^2}\,dA$$

$$z = \pm\sqrt{16-x^2-y^2}$$

$$\int_0^{2\pi}\int_2^4 2\underbrace{\sqrt{16-r^2}}_{u}\,\underbrace{r\,dr}_{-\frac{du}{2}}\,d\theta$$

$u = 16-r^2$
$du = -2r\,dr$

$$\int_2^4 2\sqrt{16-r^2}\,r\,dr = \int_{12}^{0} -\sqrt{u}\,du = \int_0^{12}\sqrt{u}\,du$$

$$= \tfrac{2}{3}u^{3/2}\Big|_0^{12}$$

$$= 16\sqrt{3}$$

$$\int_0^{2\pi} 16\sqrt{3}\,d\theta$$

$$= 16\sqrt{3}(2\pi) = \boxed{32\pi\sqrt{3}}$$

Preview:

Polar figures.

- circles centered at origin. $r = \text{constant}$
- straight lines
- circles tangent to origin