9/19/14 MATH 2850

$$(1\ \ 2\ \ 3)\begin{pmatrix}4\\5\\6\end{pmatrix} = (1\cdot4)+(2\cdot5)+(3\cdot6) = 32$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{pmatrix}7\\8\\9\end{pmatrix} = \begin{bmatrix}(1\cdot7)+(2\cdot8)+(3\cdot9)\\(4\cdot7)+(5\cdot8)+(6\cdot9)\end{bmatrix} = \begin{bmatrix}7+16+27\\28+40+54\end{bmatrix} = \begin{bmatrix}50\\122\end{bmatrix}$$

ex: $\mathbb{R}^2 \xrightarrow{g} \mathbb{R}^3 \xrightarrow{f} \mathbb{R}^1$, $f\circ g: \mathbb{R}^2 \to \mathbb{R}^1$ ← f no g $ order

$f(x,y,z) = xy + z^2$ $\qquad (f\circ g)'(u,v) = (f'\circ g)(u,v)\, g'(u,v)$

$g(u,v) = \begin{bmatrix}u^2v\\uv\\u-v\end{bmatrix}$ $\qquad f'(x,y,z) = [\,f_x(x,y,z)\quad f_y(x,y,z)\quad f_z(x,y,z)\,]$
$= [\,y \quad x \quad 2z\,]$

$$\text{outputs}\left\{\overbrace{\begin{bmatrix} & h' & \end{bmatrix}}^{\#\text{ inputs} = n}\right.$$

$f'(g(u,v)) = [\,uv \quad u^2v \quad 2(u-v)\,]$

$g'(u,v) = \begin{bmatrix}2uv & u^2\\ v & u\\ 1 & -1\end{bmatrix}$

$= [\,uv \quad u^2v \quad 2u-2v\,]\begin{bmatrix}2uv & u^2\\ v & u\\ 1 & -1\end{bmatrix}$

$= [\,uv(2uv)+(u^2v)(v)+(2u-2v)(1) \qquad (uv)(u^2)+(u^2v)(u)+(2u-2v)(-1)\,]$

$= [\,3u^2v^2+2u-2v \qquad 2u^3v-2u+2v\,] = \left[\frac{\partial f\circ g}{\partial u} \quad \frac{\partial f\circ g}{\partial v}\right]$

- compute: $(f\circ g)'(1,2)$
  $= [\,3\cdot1^2\cdot2^2+2\cdot1-2\cdot2 \qquad 2\cdot1^3\cdot2-2\cdot1+2\cdot2\,]$
  $= [\,10 \quad 6\,]$

or

$g(1,2) = \begin{bmatrix}2\\2\\-1\end{bmatrix}$ $\qquad f'(g(1,2)) = f'(2,2,-1) = (2\ \ 2\ \ -2)$

$g'(1,2) = \begin{bmatrix}4 & 1\\2 & 1\\1 & -1\end{bmatrix}$ $\quad (2\ \ 2\ \ -2)\begin{bmatrix}4 & 1\\2 & 1\\1 & -1\end{bmatrix} = [\,10 \quad 6\,]$

B

- Compute $(f \circ g)'(1,2)$
- Branch diagram:

$$\frac{dw}{du} = \frac{dw}{dx}\frac{dx}{du} + \frac{dw}{dy}\frac{dy}{du} + \frac{dw}{dz}\frac{dz}{du}$$

$$(f \circ g)'(u,v) = [\underbrace{3u^2v^2 + 2u - 2v}_{\frac{dw}{du}} \quad \underbrace{2u^3v - 2u + 2v}_{\frac{dw}{dv}}]$$

$$\frac{dw}{du} = y(2uv) + x(v) + 2z(1)$$
$$= uv\,2uv + u^2vv + 2u - 2v$$

ex: $x^2 + y^2 + z^2 = 9$ ← sphere of radius 3

y dependent — implicit differentiation

x, z independent — $\frac{dy}{dx}$ $\frac{dz}{dx}$

Diff with x:

$$2x + 2y\frac{\partial y}{\partial x} + 0 = 0 \qquad 2y\frac{dy}{dx} = -2x \qquad \frac{\partial y}{\partial x} = \frac{-x}{y}$$

Diff. with z:

$$\frac{\partial y}{\partial z} = -\frac{z}{y}$$ ← symmetry

ex: $e^{x^2+y+z^2} + \cos(xy) = 5$

x – dependent variable y, z – independent

$$\frac{\partial x}{\partial y} = e^{x^2+y+z^2}\frac{\partial}{\partial y}(x^2+y+z^2) - \sin(xy)\frac{\partial}{\partial y}(xy) = 0$$

$$= e^{x^2+y+z^2}\left(2x\frac{\partial x}{\partial y} + 1\right) - \sin(xy)\left(\frac{\partial x}{\partial y}y + x\right) = 0$$

$$\left[e^{x^2+y+z^2}2x - \sin(xy)y\right]\frac{\partial x}{\partial y} + \left[e^{x^2+y+z^2}\cdot 1 - \sin(xy)\cdot x\right] = 0$$

Continued:

$$\frac{dx}{dy} = \frac{-\left[e^{x^2+y+z^2} - x\sin(xy)\right]}{e^{x^2+y+z^2}2x - y\sin(xy)\Big]}$$